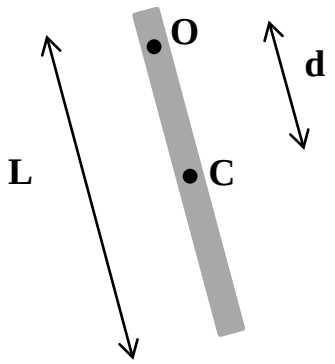
# Oscillation examples

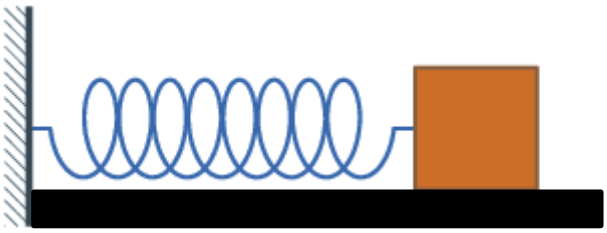
**Example 1**




A stick of length L and mass m oscillates about an axis O (as shown above). The center of mass C of the stick is located a distance d away from point O. Initially the stick is released from rest at an angle $\theta_0$ from the vertical.

a) Draw an extended free body diagram of the stick.
b) Derive the equation of motion.
c) What is the solution of the equation of motion if the initial angle $\theta_0$ is small (i.e. < 10°)? Sketch the graph θ(t).
d) What are the amplitude, period, frequency, angular frequency, and phase constant of the motion?

**Example 2**

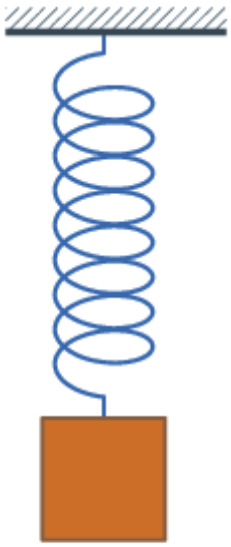


A mass m = 1kg is attached to a spring on a flat smooth surface. Initially, the spring is at its rest position (i.e. not stretched nor compressed) and the mass is given an initial velocity $v_0$ = 1 m/s to the left. The spring constant is k = 100 N/m.

a) Derive the equation of motion.
b) What is the solution of the equation of motion?
c) What are the amplitude, period, frequency, angular frequency, and phase constant of the motion?
d) At which location(s) is the velocity of the mass a maximum? A minimum?
e) At which location(s) is the acceleration of the mass a maximum? A minimum?
f) Plot x(t), v(t) and a(t) on the same graph.
g) Find the kinetic energy, potential energy and mechanical energy as a function of time.
h) For different initial conditions, the equation is found to be $x(t) = 2cos(\omega t + 60°)cm$. Find the intial conditions x(t=0) and v(t=0).

**Example 3**

Repeat example 2 with the mass hanging from the ceiling. Initially the mass is at rest and is given an initial velocity $v_0 = 1$ m/s directed up.



**Example 4: circular motion and simple harmonic motion**

A mass m is hanging on a spring of spring constant k. The mass oscillates up and down with amplitude A. A disk is rotating next to the mass so that the two points A and B are always at the same height. What is the radius of the disk? What is the angular velocity of the disk (direction and magnitude)?