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## Addition of Vectors



## Purpose:

In this lab we will explore physically, geometrically, and algebraically the addition of vectors.

We will do this by taking advantage of a special case of Newton's second law, namely that the sum of the forces acting on an object in static equilibrium is zero. Since forces are vectors, we will practice doing vector additions.

First, we measure the forces acting on an object in equilibrium (a ring). Then we check that adding the forces as vectors gives zero. We do this in two ways: graphically and analytically (i.e. using vector components).

## Equipment:

- Protractor
- Ruler
- String
- 1 ring
- 4 mass hangers
- Set of slotted masses
- Force table
- 4 pulleys


## What to turn in next week:

1. These lab instructions filled out correctly and completely. No typing necessary but make it is clear and legible. Turn in one copy per group. Every group member should check the work.
2. Attached graphs showing clearly the graphical vector addition that you have carried out.

Notes on the graphs:

- Graphs need to be to scale, not sketched!
- Everything should be labeled, i.e. if you draw an angle of 20 degrees, indicate that in your graph, if you draw and arrow representing $\vec{F}_{1}+\vec{F}_{2}$, label the arrow, etc.
- Include a scale, e.g., $3 \mathrm{~cm}=1 \mathrm{~N}$, or whatever is a good scale to fit your paper.
- The graphs should fill the page reasonably well, don't make them too small!
- For all vectors, give BOTH the length in cm (or inches) AND (converted with your scale) in Newtons.
- If a quantity is a vector, make sure to put an arrow on top of the symbol, if you are referring to the magnitude, use either absolute value signs or no arrow.


## Note on tables in this lab:

In your tables you will find the words 'calc', 'exp', and 'graph'. These are supposed to help you fill in the correct information.

1. "calc." stands for calculated. For example, if the mass is 0.1 kg , the gravitational force or weight due to the mass is calculated using weight $=\mathrm{mg}$, or $0.1 \mathrm{~kg} * 9.8 \mathrm{~m} / \mathrm{s}^{2}=0.98 \mathrm{~N}$.
2. "exp" stands for experimental. These values you get directly from measurements. You should not have calculated these.
3. "graph" stands for graphical. The values in these boxes should not be the analytical solution. They should come directly from your graphical solution.

## Addition of Vectors

## Part A: Three forces on a ring in equilibrium

## Part A1: Experimental Data

a) Place a pulley at the $20.0^{\circ}$ mark on the force table and place a total of 100 . grams on the end of the string. Be sure to include the 50.0 grams of the mass hanger in the total (i.e. hanger + additional mass = 100 g).

Calculate the magnitude of the force produced by the mass and hanger (in Newtons. Use $\mathrm{F}=\mathrm{mg}$ and $\mathrm{g}=9.80 \mathrm{~m} / \mathrm{s}^{2}$ ). Work to three significant figures for this and for all other calculations of forces in this lab. Record the value of this force as $\vec{F}_{1}$ in Table A1.
b) Place a second pulley at the $90.0^{\circ}$ mark on the force table and place a total of 200 g on the end of the string. Calculate the force produced and record this force as $\vec{F}_{2}$ in Table A1.
c) Determine by trial and error the amount of mass needed and the angle at which it must be placed in order to bring the ring to equilibrium. Make sure that the ring is centered on the force table and is not touching the peg in the middle. Jiggle the ring slightly to make sure that it is not stuck. Be sure that all the strings lie along a line which passes through the center of the ring.
d) For this third mass, calculate the amount of force produced and from the force table markings determine the angle at which the force acts. Record this force as $\vec{F}_{3}$ in Table A1 (on the next page).

Table A1 - Experimental Results

| Force | $\underline{\text { Mass (kg) }}$ | $\begin{aligned} & \text { Force (N) } \\ & \text { Magnitude } \end{aligned}$ | $\begin{gathered} \text { Force } \\ \text { Direction } \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| $\vec{F}_{1}$ | 0.100 | calc. | $20.0^{\circ}$ |
| $\vec{F}_{2}$ | 0.200 | calc. | $90.0^{\circ}$ |
| $\vec{F}_{3}$ | exp. | calc. | exp. |

## Part A2: Graphical Result

As a group, find the resultant of the two applied forces $\vec{F}_{1}$ and $\vec{F}_{2}$ by a scaled, graphical construction using the parallelogram method (place the vectors tail to tail). Use the graph paper attached to this lab (or your own engineering paper).

Question 1: How is $\vec{F}_{3}$ related to that resultant?

Draw and label $\vec{F}_{3}$ on the graph as well.
Record your results in Table A2 below.
Table A2-Graphical Solution

| $\underline{\text { Force }}$ | Mass (kg) | Force (N) <br> Magnitude | Force <br> Direction |
| :---: | :--- | :--- | :--- |
| $\vec{F}_{1}$ | 0.100 | calc. | $20.0^{\circ}$ |
| $\vec{F}_{2}$ | 0.200 | calc. | $90.0^{\circ}$ |
| $\vec{F}_{1}+\vec{F}_{2}$ | calc. | graph | graph |
| $\vec{F}_{3}$ | calc. | calc. | calc. |

## Part A3: Analytical Result

Using trigonometry, calculate the components of $\vec{F}_{1}$ and $\vec{F}_{2}$ and record them in Table A3. Define the x -axis as the line going through the $0^{\circ}$ mark and the y -axis the line going through the $90^{\circ}$ mark.

Add the components and determine the resultant $\vec{F}_{1}+\vec{F}_{2}$, both magnitude and direction.
Fill out the remainder of the Table A3.
Table A3-Analytical Solution

| Force | $\underline{\text { Mass (kg) }}$ | Force (N) <br> Magnitude | Force <br> Direction | $\underline{\underline{\mathbf{x}-c o m p ~(N) ~}}$ | y-comp (N) |
| :---: | :--- | :--- | :---: | :--- | :--- |
| $\vec{F}_{1}$ | 0.100 | calc. | $20.0^{\circ}$ | calc. | calc. |
| $\vec{F}_{2}$ | 0.200 | calc. | $90.0^{\circ}$ | calc. | calc. |
| $\vec{F}_{1}+\vec{F}_{2}$ | calc. | calc. | calc. | calc. | calc. |
| $\vec{F}_{3}$ | calc. | calc. | calc. | calc. | calc. |

## Part A4: Errors

a) Calculate the percentage error between the experimental value and the analytical calculation for the magnitude of $\vec{F}_{3}$. Show your math.
b) Calculate the percentage error between the graphical result and the analytical calculation for the magnitude of $\vec{F}_{3}$. Show your math.
c) Are the errors large or small? Are your three results for $\vec{F}_{3}$ roughly the same?

## Part B: Four forces on a ring in equilibrium

## Part B1: Experimental Data

a) Place a pulley at 30.0 degrees with 150 . grams on it, one at 100 . degrees with 200. grams on it and one at 145 degrees with 100. grams on it.
b) Calculate the force produced by those masses and record them as $\vec{F}_{4}, \vec{F}_{5}$, and $\vec{F}_{6}$ respectively in Table B1.
c) As in part A, by trial and error add the appropriate mass configuration to give a net force of zero. Record the added force as $\vec{F}_{7}$ in Table B1 (on the next page).

Question 2: Could all four pulleys be placed in the same quadrant or in two adjacent quadrants and still be in equilibrium? Explain using a sketch of vectors.

Question 3: The forces used in this experiment are the weights of known masses, that is, the forces exerted on these masses by gravity. Bearing this in mind, explain the function of the pulleys.

## Table B1 - Experimental Results

| Force | Mass (kg) | Force (N) <br> Magnitude | Force <br> Direction |
| :---: | :---: | :--- | :---: |
| $\vec{F}_{4}$ | 0.150 | calc. | $30.0^{\circ}$ |
| $\vec{F}_{5}$ | 0.200 | calc. | $100.0^{\circ}$ |
| $\vec{F}_{6}$ | 0.100 | calc. | $145.0^{\circ}$ |
| $\vec{F}_{7}$ | exp. | calc. | exp. |

## Part B2: Graphical Result

As a group, find the resultant of the two applied forces $\vec{F}_{4}, \vec{F}_{5}$ and $\vec{F}_{6}$ by a scaled, graphical construction using the tail-to-tip method. Use the graph paper attached to this lab (or your own engineering paper).

Draw and label $\vec{F}_{7}$ on the graph as well.
Record your results in Table B2.
Table B2-Graphical Solution

| Force | Mass (kg) | Force (N) <br> Magnitude | Force <br> Direction |
| :---: | :--- | :--- | :--- |
| $\vec{F}_{4}$ | 0.150 | calc. | $30.0^{\circ}$ |
| $\vec{F}_{5}$ | 0.200 | calc. | $100.0^{\circ}$ |
| $\vec{F}_{6}$ | 0.100 | calc. | $145.0^{\circ}$ |
| $\vec{F}_{4}+\vec{F}_{5}+\vec{F}_{6}$ | calc. | graph | graph |
| $\vec{F}_{7}$ | calc. | calc. | calc. |

## Part B3: Analytical Result

Using trigonometry, calculate the components of $\vec{F}_{4}, \vec{F}_{5}$ and $\vec{F}_{6}$ and record them in Table B3. Define the x -axis as the line going through the $0^{\circ}$ mark and the y -axis the line going through the $90^{\circ}$ mark.

Add the components and determine the resultant $\vec{F}_{4}+\vec{F}_{5}+\vec{F}_{6}$, both magnitude and direction.

Fill out the remainder of the Table B3.
Table B3-Analytical Solution

| $\underline{\text { Force }}$ | $\underline{\text { Mass (kg) }}$ | Force (N) <br> Magnitude | Force <br> Direction | $\underline{\underline{\text { x-comp (N) }}}$ | $\underline{\text { y-comp (N) }}$ |
| :---: | :--- | :--- | :---: | :--- | :--- |
| $\vec{F}_{4}$ | 0.150 | calc. | $30.0^{\circ}$ | calc. | calc. |
| $\vec{F}_{5}$ | 0.200 | calc. | $100.0^{\circ}$ | calc. | calc. |
| $\vec{F}_{6}$ | 0.100 | calc. | $145.0^{\circ}$ | calc. | calc. |
| $\vec{F}_{4}+\vec{F}_{5}+\vec{F}_{6}$ | calc. | calc. | calc. | calc. | calc. |
| $\vec{F}_{7}$ | calc. | calc. | calc. | calc. | calc. |

## Part B4: Errors

a) Calculate the percentage error between the experimental value and the analytical calculation for the magnitude of $\vec{F}_{7}$. Show your math.
b) Calculate the percentage error between the graphical result and the analytical calculation for the magnitude of $\vec{F}_{7}$. Show your math.
d) Are the errors large or small? Are your three results for $\vec{F}_{7}$ roughly the same?

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