Multiple choice questions [100 points]
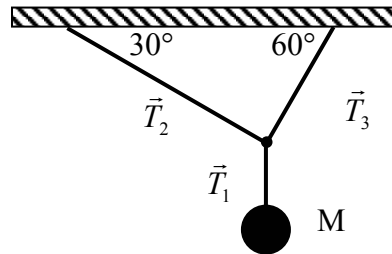
Answer all of the following questions. Read each question carefully. **Fill the correct bubble on your scantron sheet**. Each correct answer is worth 4 points. Each question has exactly one correct answer.

1. As shown in the figure, a mass M is hanging by three massless strings from the ceiling of a room. Let $\vec{T}_1$, $\vec{T}_2$, and $\vec{T}_3$ denote the tensions in the 3 strings.
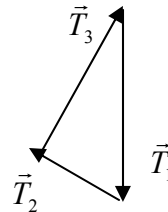

Choose one among the following

A. $\left|\vec{T}_2\right| > \left|\vec{T}_3\right|$

B. $\left|\vec{T}_2\right| \underline{\ <\ } \left|\vec{T}_3\right|$ Write that the node (where the 3 strings meet) is in equilibrium:
$\overline{T_2cos30 = T_3cos60}$

C. $\left|\vec{T}_2\right| = \left|\vec{T}_3\right|$
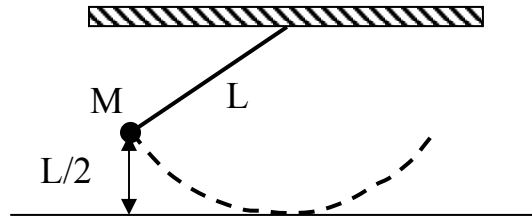
2. Still referring to the situation described in question 1, choose one among the following:

A. $\left|\vec{T}_1\right| > \left|\vec{T}_2\right| + \left|\vec{T}_3\right|$

   $\left|\vec{T}_1\right| < \left|\vec{T}_2\right| + \left|\vec{T}_3\right|$ In a rectangle triangle, the hypothenuse is less than the sum of the 2 sides of the right angle.

B.

C. $\left|\vec{T}_1\right| = \left|\vec{T}_2\right| + \left|\vec{T}_3\right|$

3.

As shown in the figure, a small sphere of mass M attached to a massless string of length L is released at height L/2 and allowed to swing back and forth. Ignore air resistance.
Let your system consist of the sphere at the end of the string. For this system:
A.   only kinetic energy is conserved
B.   <u>only total mechanical energy is conserved</u>
C.   only total mechanical energy and linear momentum is conserved
D.   each of total mechanical energy, linear momentum, and angular momentum is conserved.
E.   each of total mechanical energy and the sum of linear plus angular momentum is conserved.

4.   A man turns with an angular velocity on a rotation table, holding two equal masses at arms' length. If he drops the two masses without moving his arms, his angular velocity
A.   decreases
B.   <u>remains the same</u>
C.   increases

5.   A wheel is rotating freely with an angular speed of 20 rad/s on a shaft whose moment of inertia is negligible. A second identical wheel, initially at rest, is suddenly coupled to the same shaft. The angular speed of the coupled wheels is
A.   <u>**10 rad/s**</u>(angular momentum is conserved: $20I = 2I\omega$)
B.   **14 rad/s**
C.   **20 rad/s**
D.   **28 rad/s**
E.   **40 rad/s**

Questions 6 through 13 all refer to the same problem.

A small ball of mass m=100 g hangs by a massless inextensible string from the ceiling of a railway car. You may treat the ball as a point particle.
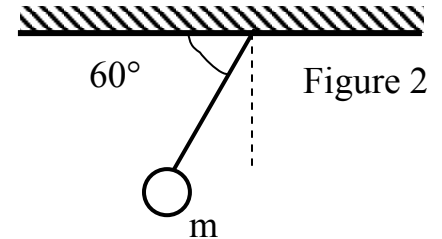
At the instant shown in figure 1, the train is at rest in the station.
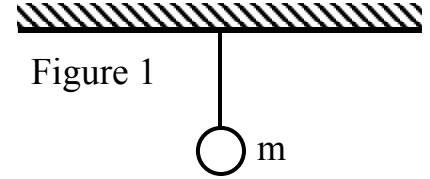
Figure 1

6. The tension in the string is

        m

    A. <u>0.98 N</u>
    B. 9.8 N
    C. 0 N
    D. Other
    E. Can't tell. There is not enough information.

Sometime later, the train is moving with <u>decreasing speed</u>. At the instant shown in figure 2, the speed is v_T and the ball is observed to hang as shown.
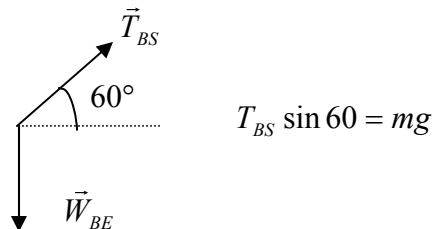
7. The velocity of the train is directed:

    60°   Figure 2

    A. To the right
    B. <u>To the left</u> (The acceleration $\vec{a}$ is to the right. Thus, the change of velocity $\Delta\vec{v} = \vec{a}\Delta t$ is to the right. If the speed is decreasing, $\Delta\vec{v}$ and $\vec{v}$ have opposite directions).
    C. Can't tell. There is not enough information.
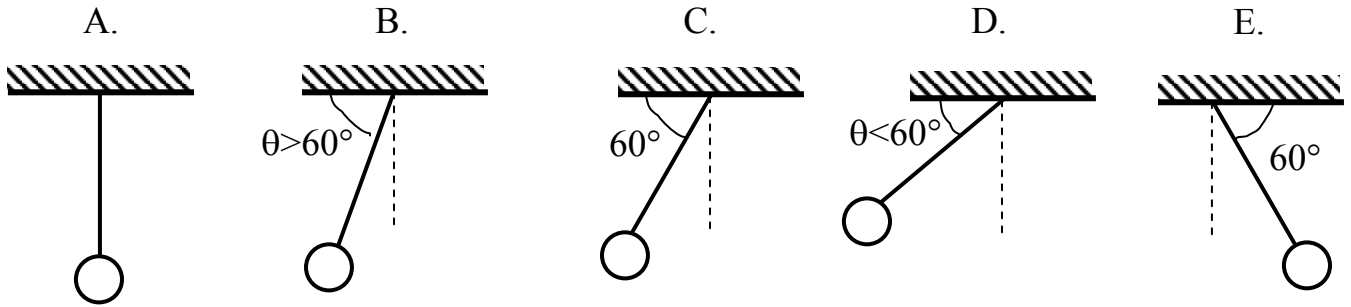
        m

8. The tension in the string is

$\vec{T}_{BS}$

    A. 0.98 N
    B. <u>1.13 N</u>   60°
    C. 0.849 N $T_{BS}\sin 60 = mg$
    D. 1.96 N
    E. Other

$\vec{W}_{BE}$

The train continues to move with *constant acceleration.*

These sketches are for questions 9, 10 and 11.

A.   B.   C.   D.   E.

$\theta > 60°$ $60°$ $\theta < 60°$ $60°$

9. Which of the sketches above best represents the ball when the speed of the train is $0.5v_T$?

Use $T_{BS} \cos\theta = ma$ and $T_{BS} \sin\theta = mg$ to find $\tan\theta = \dfrac{g}{a}$

If $\vec{a}$ doesn't change, θ remains the same.
C

10. Which of the sketches above best represents the ball when the train is reversing
direction and has 0 speed?
$\vec{a}$ remains the same: C

Elsewhere on the train, a second ball of mass *2m* hangs from a string of the same length
as that of the first ball.

11. Which of the sketches above best represents ball 2 at the instant depicted in figure 2?

$\tan\theta = \dfrac{g}{a}$ doesn't' depend on the mass: C

Suppose that at the instant depicted by figure 2, the string holding the ball of mass *m*
breaks.

12. Which of the following best represents the trajectory of the ball as seen by an
observer on the ground (i.e. in the reference frame of the ground)?

A.   B.   C.   D.   E.
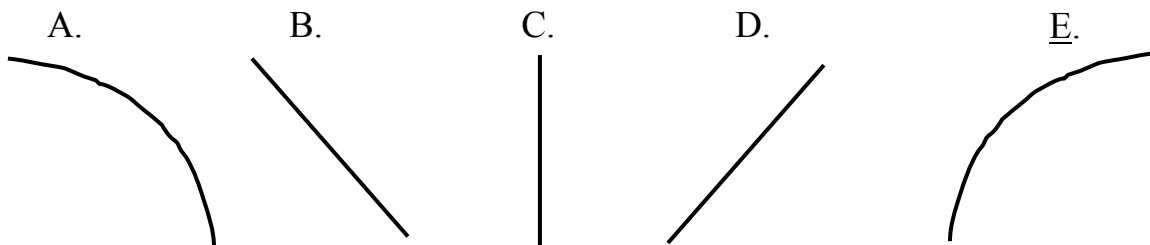
Free fall with a horizontal initial velocity directed to the left.

13. The point of the floor that is directly below the ball when the string breaks is marked with an 'X'. When the ball falls, it lands

    A. on the X
    B. to the left of the X (The train slows downs as the ball falls).
    C. to the right of the X
    D. Can't tell. There is not enough information.

Questions 14 through 18 all refer to the same problem.

A race car starts at rest on a flat circular track with a radius of 100m. With uniformly accelerated motion ($a_{//} = \dfrac{d|\vec{v}|}{dt} = $ constant), the car completes one lap around the track in 60 seconds. The car has a mass of 500 kg.

14. As viewed from the center of the track, what is the rotational acceleration of the race car?

    A. 1.7×10^{-3} rad/s^2

    B. 3.5×10^{-3} rad/s^2 ($\alpha = ra_{//} = $ constant, thus $\theta = \dfrac{1}{2}\alpha t^2$. $\alpha = \dfrac{4\pi}{60^2}$)

    C. 0.10 rad/s^2
    D. 0.17 rad/s^2
    E. 0.21 rad/s^2

15. As viewed from the center of the track, what angle has the race car traveled after 20 seconds? (The answers are expressed in radians)
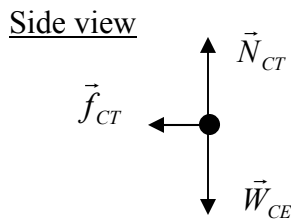
    A. π/20
    B. π/9

    C. 2π/9 (use $\theta = \dfrac{1}{2}\alpha t^2$ for t=20s)

    D. π/3
    E. 2π/3

16. What is the average speed of the car during the first lap around the track?

    A. 0
    B. 1.67 m/s
    C. 3.77 m/s
    D. 5.24 m/s

    E. 10.5 m/s ($v_{avg} = \dfrac{2\pi r}{t_{1lap}} = \dfrac{200\pi}{60}$)

17. Let f denote the magnitude of the frictional force of the road on the car, and let W denote the magnitude of the weight of the car. As the car drives around the track, what can you say about the ratio f/W?

   A. f/W increases
   B. f/W decreases
   C. f/W stays the same
   D. Can't say anything without any more information.

Top view

Race track

$\vec{f}_{CT}$

Car (moving in counterclockwise direction)

Side view

$\vec{N}_{CT}$

$\vec{f}_{CT}$

$\vec{W}_{CE}$
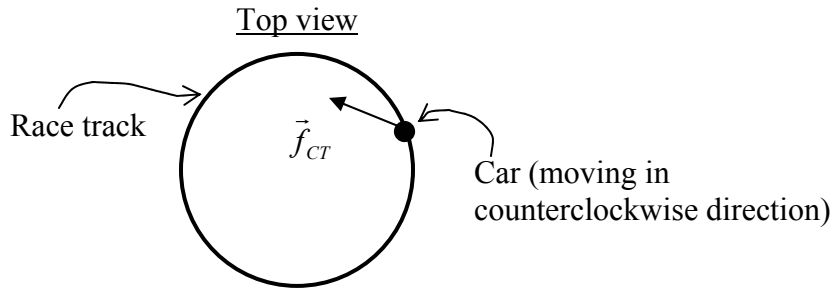
$$\vec{f}_{CT} = m\vec{a} = m\vec{a}_{//} + m\vec{a}_{\perp}$$

$$\left|\vec{f}_{CT}\right| = m\sqrt{(r\alpha)^2 + \left(\frac{v^2}{r}\right)^2}$$

As the car drives around the track, v increases. Thus, f_{CT} increases

18. Later, during a race, the race car will travel at *constant speed* around the track. The driver needs to know the absolute maximum speed that he *might* be able to drive around the track without sliding. Fortunately, he hires you to calculate this. Your answer should be:
   A. The speed is 3×10^8 m/s. Wahoo!
   B. As your tires get more and more frictional, you can drive faster and faster, until you are limited by the performance of your car's engine.
   C. Even if you have the best tires, you cannot possibly drive faster than 16 m/s on that track, and you might not even be able to go that fast without sliding.
   D. Even if you have the best tires, you cannot possibly drive faster than 31 m/s on that track, and you might not even be able to go that fast without sliding.
   $$f_{CT} \le \mu_s N_{CT} = \mu_s W_{CE}$$

Since $\mu_S \leq 1$, $f_{CT} \leq W_{CE}$

Also, $f_{CT} \geq m\dfrac{v^2}{r}$. It follows that $m\dfrac{v^2}{r} \leq W_{CE}$. That is $v \leq \sqrt{rg} = 31.3\text{m/s}$

E. Even if you have the best tires, you cannot possibly drive faster than 51 m/s on that track, and you might not even be able to go that fast without sliding.

Questions 19 through 21 all refer to the same problem.

You are standing in the middle of a road, and a truck is driving directly toward you at a constant speed of 10m/s. To get the driver's attention, you decide to throw a perfectly elastic rubber ball with a mass of 0.1 kg directly at the front of the truck. The front of the truck is frictionless, perfectly vertical, and flat.
The ball has a velocity of $5.0\,\hat{x}$ m/s just before it strikes the truck ($\hat{x}$ is a horizontal unit vector). Assume that the collision is perfectly elastic.

19. Immediately after the collision, the horizontal velocity of the ball is approximately

    A. $-5.0\,\hat{x}$ m/s

    B. $-10.0\,\hat{x}$ m/s

    C. $-15.0\,\hat{x}$ m/s

    D. $-20.0\,\hat{x}$ m/s

    E. $\underline{-25.0\,\hat{x}\ \text{m/s}}$

The collision is elastic thus: $\vec{v}_b^{\,f} - \vec{v}_t^{\,f} = -\left(\vec{v}_b^{\,i} - \vec{v}_t^{\,i}\right)$

The velocity of the truck will not change significantly before and after the collision since $M_t \gg M_b$.

Thus: $\vec{v}_b^{\,f} = \vec{v}_t^{\,f} + \vec{v}_t^{\,i} - \vec{v}_b^{\,i} \approx 2\vec{v}_t^{\,i} - \vec{v}_b^{\,i} = 2(-10\hat{x}) - 5\hat{x}$

20. What is the impulse received by the ball during the collision?

    A. $\underline{-3.0\,\hat{x}\ \text{kg m/s}}$

    B. $-2.5\,\hat{x}$ kg m/s

    C. $-2.0\,\hat{x}$ kg m/s

    D. $-1.5\,\hat{x}$ kg m/s

    E. $-1.0\,\hat{x}$ kg m/s

The impulse is equal to the change of momentum:

$\vec{I} = \Delta\vec{p} = M_b\vec{v}_b^{\,f} - M_b\vec{v}_b^{\,i} = 0.1(-25\hat{x} - 5\hat{x})$

21. As viewed in the truck driver's reference frame, the absolute magnitude of the horizontal velocity of the ball after the collision is

    A. the same as before the collision

    B. smaller than before the collision

    C. greater than before the collision

    D. cannot be determined from the information given.

The collision is elastic thus:

$\vec{v}_b^{\,f} - \vec{v}_t^{\,f} = -\left(\vec{v}_b^{\,i} - \vec{v}_t^{\,i}\right)$

Questions 22 through 25 all refer to the same problem.

A uniform board has length $8x$, width $2x$ (x is an unknown distance), and unknown mass m. A piece of clay of mass M is placed on the board as shown. The board with clay attached is found to balance when placed on a frictionless pivot as shown.

22. How does the mass of the board m compare to the mass of the piece of clay M?
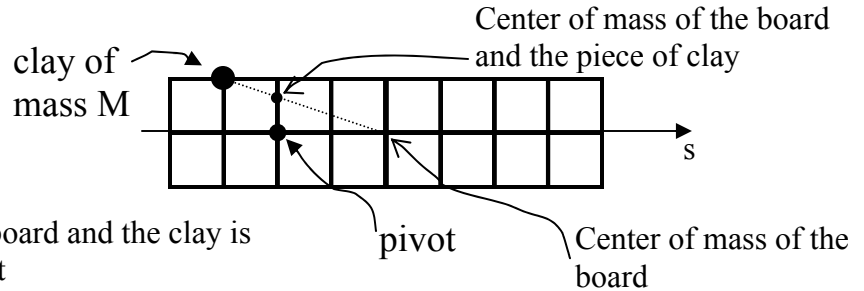
A. $m = M$

B. $m = M/2$
   The center of mass of the board and the clay is located just above the pivot
   Take an axis (labeled s) with its origin at the pivot:
   $$Ms_{CM\,clay} + ms_{CM\,board} = (M+m)s_{CM\,board+clay}$$
   $$-Mx + 2mx = 0$$

C. $m = 2M$

D. Can't say anything without any more information.

23. Which of the following best describes the board?
   A. At rest in stable equilibrium
   B. At rest in unstable equilibrium (The center of mass is above the pivot)
   C. Not in equilibrium. It will tip down to the right.
   D. Not in equilibrium. It will tip down to the left.
   E. There is not enough information to determine.

24. The rotational moment of inertia of the board *alone* about the pivot is I_b. What is the rotational moment of inertia of the board and the piece of clay about the pivot?
   A. $I_b + 2Mx^2$ (The moment of inertia of the piece of clay is $2Mx^2$)
   B. $I_b + Mx^2$
   C. $I_b + \sqrt{2}\,Mx^2$
   D. $I_b + \sqrt{3}\,Mx^2$
   E. $2I_bMx^2$

25. Later, you are told that M=10 g and the dimensions of the board are 40cm by 10cm. For fun, you start spinning the system around the pivot with a constant rotational velocity of 1rev/sec. The rotational kinetic energy of the *piece of clay* around the pivot is approximately:
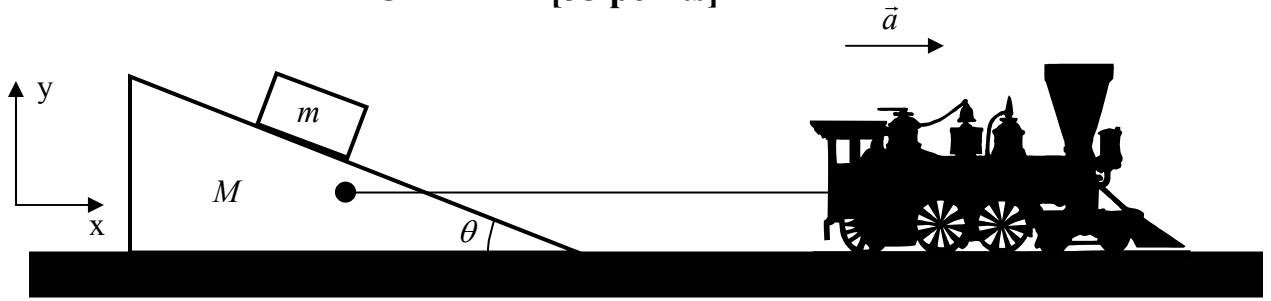   **A.** $2.5 \times 10^{-5}\,J$
   B. $5.0 \times 10^{-4}\,J$
   C. $1.0 \times 10^{-3}\,J$ ($KE = \dfrac{1}{2}I_{clay}\omega^2 = \dfrac{1}{2}(2Mx^2)\omega^2 = 0.01 \times (0.05)^2 \times (2\pi)^2$)
   **D.** $250\,J$
   E. $9870\,J$

PROBLEM 1 [35 points]

$\vec{a}$

y

m

M

x

θ

As shown in the figure, a toy train is pulling a wedge of mass M across a horizontal tabletop. A block of mass m is resting on the inclined surface of the wedge; let θ be the angle that the incline makes with the horizontal. There is <u>no friction</u> between the block and the wedge and there is <u>no friction</u> between the wedge and the table. Assume that the massless string that connects the train to the wedge is exactly horizontal. As you watch the experiment, you notice that **the train is accelerating** and you notice that **the block remains at the same height on the incline** (in other words **the acceleration of the block is horizontal**).

1). [5 pts] The magnitude of the net acceleration vector of the block is (no explanation necessary)
   a. greater than the magnitude of the acceleration vector of the wedge.
   b. less than the magnitude of the acceleration vector of the wedge.
   c. <u>equal to the magnitude of the acceleration vector of the wedge.</u> (if not, the block would slide up or down the wedge).

2). [5 pts] In the boxes below, draw free body diagrams for the block and the wedge. Label each force vector.

θ $\vec{N}_{BW}$

$\vec{W}_{BE}$

Block

$\vec{N}_{WT}$

$\vec{T}_{WS}$

θ

$\vec{N}_{WB}$ $\vec{W}_{WE}$
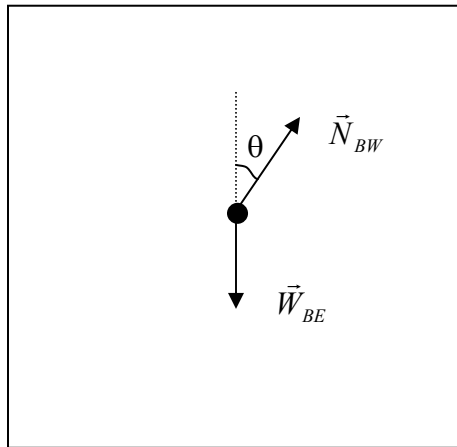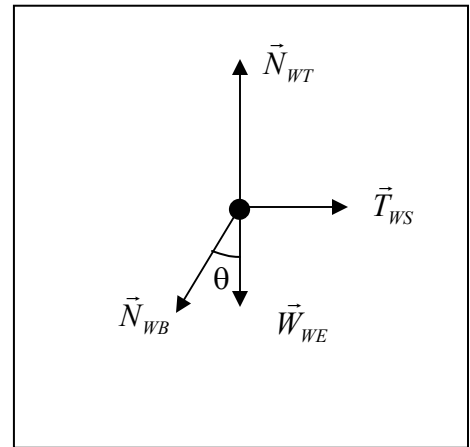
Wedge

3). [10 pts] In the space below, write the x-component and y component equations from Newton's second law for the block. Clearly indicate if any acceleration component is zero. Use notation consistent with the labeling of the forces in your free body diagrams.

$$\vec{W}_{BE} + \vec{N}_{BW} = m\vec{a}$$

$\vec{a}$ is along $\hat{x}$

Along x: $N_{BW} \sin\theta = ma$

Along y: $N_{BW} \cos\theta - mg = 0$

4). [10 pts] In the space below, write the x-component and y component equations from Newton's second law for the wedge. Clearly indicate if any acceleration component is zero. Use notation consistent with the labeling of the forces in your free body diagrams.

$$\vec{T}_{WS} + \vec{N}_{WT} + \vec{W}_{WE} + \vec{N}_{WB} = M\vec{a}$$

Along x: $T_{WS} - N_{WB} \sin\theta = Ma$

Along y: $N_{WT} - Mg - N_{WB} \cos\theta = 0$

5). [5 pts]Solve for the magnitude of the tension in the string in terms of M, m, θ, and g. Show your work.

$N_{WB} = N_{BW}$ (3$^{\text{rd}}$ law)

From 3): $a = g\tan\theta$ and $N_{BW} = \dfrac{mg}{\cos\theta}$

And from 4)
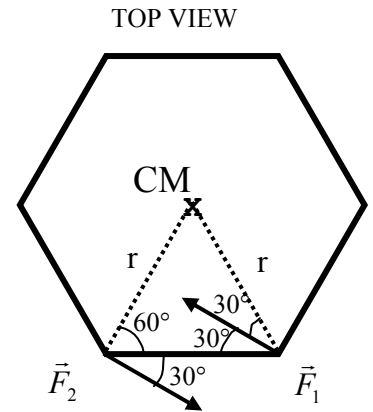
$T_{WS} = Mg\tan\theta + mg\tan\theta$

$T_{WS} = (M+m)g\tan\theta$

TOP VIEW

PROBLEM 2[25 points]

A hexagonal block is at rest upon a level frictionless surface, as shown in the top view diagram at right.
Note: The block is *not* attached to the surface and is free to move.

CM

r   r

$30°$

$60°$ $30°$

$30°$

$\vec{F}_2$ $\vec{F}_1$

A [16 pts] Two forces $\vec{F}_1$ and $\vec{F}_2$, equal in magnitude and opposite in direction, are exerted on the block as indicated.

a. [8 pts] Find the direction of $\vec{\alpha}$, the block's angular acceleration vector about its center of mass (CM). If $\vec{\alpha}=0$, then indicate that explicitly. Explain.

Torque for $\vec{F}_1$: $rF_1 \sin(150°)$ into the page

Torque for $\vec{F}_2$: rF_2 out of the page

The net torque $\vec{\tau}$ is out of the page

Since $\vec{\tau} = I\vec{\alpha}$, $\vec{\alpha}$ is out of the page (the block starts rotating counterclockwise).

b. [8 pts] Find the direction of $\vec{a}_{CM}$, the acceleration vector of the block's center of mass. If $\vec{a}_{CM}=0$, then indicate that explicitly. Explain.

$$\vec{F}_1 + \vec{F}_2 = m\vec{a}_{CM} = 0$$
$$\vec{a}_{CM} = 0$$

B [9 pts] In this part of the problem consider the torque produced by $\vec{F}_1$ (taken about the center of mass) when it is exerted at each of the points a, b, c, and d labeled in the diagram below.

CM   CM   CM   CM

a   b   c   d
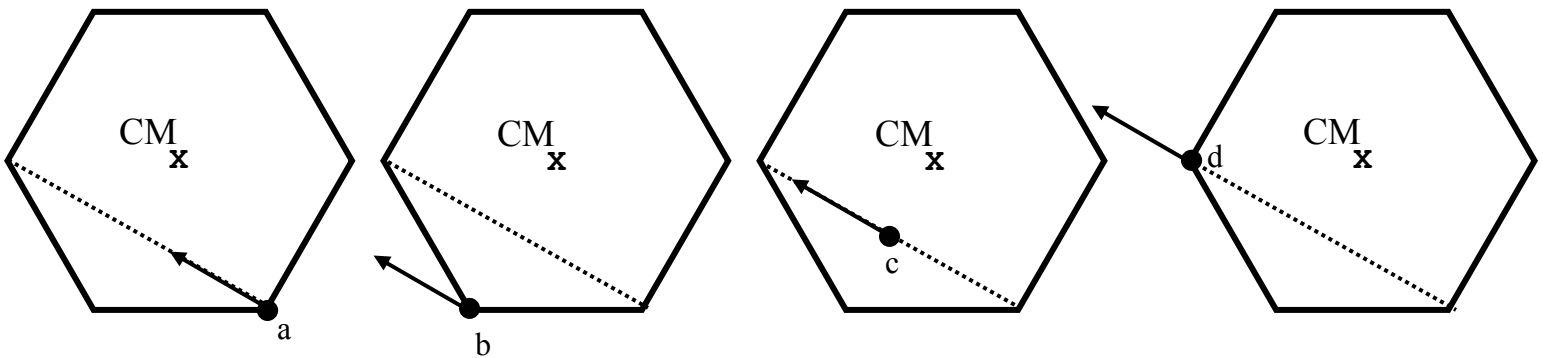
Rank the magnitudes of these torques from largest to smallest. Explain the reasoning you

used to determine your ranking.

Use $\quad \vec{\tau} = \vec{r} \times \vec{F}$, $\left|\vec{\tau}\right| = rF\sin\theta$

$$\left|\vec{\tau}_a\right| = rF_1\sin(150) = \frac{1}{2}rF_1$$

$$\left|\vec{\tau}_b\right| = rF_1\sin(90) = rF_1$$

$$\left|\vec{\tau}_c\right| = \frac{r}{2}F_1\sin(90) = \frac{1}{2}rF_1$$

$$\left|\vec{\tau}_d\right| = \frac{r}{2}F_1\sin(30) = \frac{1}{2}rF_1$$

$$\left|\vec{\tau}_a\right| = \left|\vec{\tau}_c\right| = \left|\vec{\tau}_d\right| < \left|\vec{\tau}_b\right|$$