

(Last)

(First)

Multiple choice questions [60 points]

Answer all of the following questions. Read each question carefully. **Fill the correct bubble on your scantron sheet.** Each question has exactly one correct answer. All questions are worth the same amount of points.

1. Susana ascends a mountain via a short, steep trail. Sean ascends the same mountain via a long, gentle trail. Which of the following statements is true?
 - A. Susana gains more gravitational potential energy than Sean.
 - B. Susana gains less gravitational potential energy than Sean.
 - C.** Susana gains the same gravitational potential energy as Sean.
 $\Delta U = mgh$ same h for both (note that we need to assume that Sean and Susana have the same mass). The best answer (not listed) would be: to compare gravitational potential energies, we must know the masses of Sean and Susana. This question was counted correct for everyone.
 - D. To compare gravitational potential energies, we must know the height of the mountain.
 - E. To compare gravitational potential energies, we must know the lengths of the two trails.

(Last) (First)

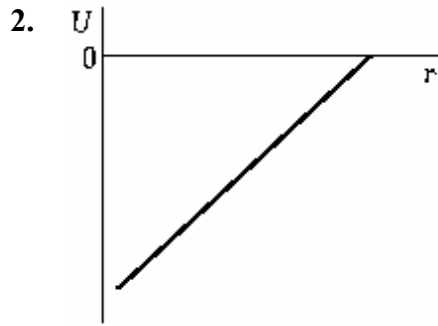


Figure A

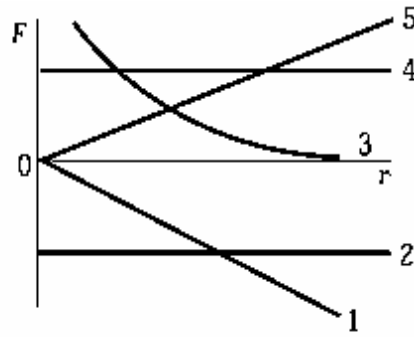


Figure B

When the potential energy $U(r)$ is given as in Figure A, then the force is given in Figure B by curve

A. 1

B. 2

$F = -\frac{dU}{dr}$ Since the graph of $U(r)$ is a straight line with a positive slope, F is constant and negative.

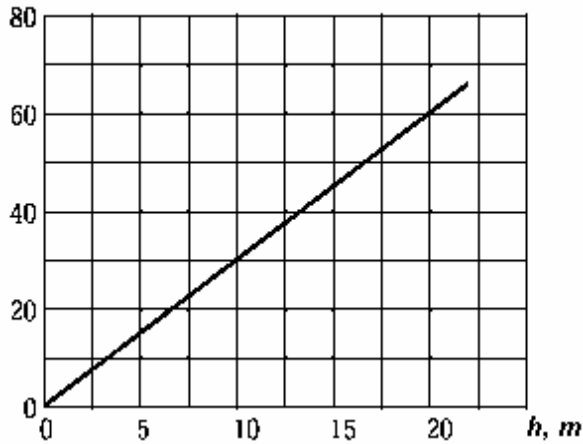
C. 3

D. 4

E. 5

(Last)

(First)

3. U, J 

The graph shows a plot of the gravitational potential energy U of a 1-kg body as a function of its height h above the surface of a planet. The acceleration due to gravity at the surface of the planet is

- A. 0 m/s^2
- B. 9.8 m/s^2
- C. 6 m/s^2
- D. 3 m/s^2**

For instance, the change of gravitational potential energy for the 1kg mass between 0 and 20 m is

$$\Delta U = U(20) - U(0) = 60 \text{ J} \quad \text{also} \quad \Delta U = mgh = 20g \Rightarrow g = \frac{60}{20}$$

- E. None of these is correct.

(Last)

(First)

4. Two unequal masses hang from either end of a massless cord that passes over a frictionless pulley. Which of the following is true about the gravitational potential energy (U) and the kinetic energy (K) of the system consisting of the two masses after the masses are released from rest?

A. $\Delta U < 0$ and $\Delta K > 0$

Only conservative forces do work (namely the weight of the two masses). The work by the tensions on both masses cancel out.

The mechanical energy of the system (made of the 2 masses + Earth) is constant: $K+U=\text{constant}$

Since the velocity of both masses increases, $\Delta K > 0$. And since $\Delta K = -\Delta U$, $\Delta U < 0$

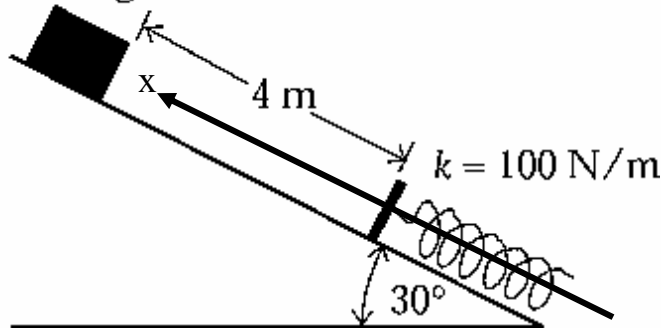
B. $\Delta U = 0$ and $\Delta K > 0$

C. $\Delta U < 0$ and $\Delta K = 0$

D. $\Delta U = 0$ and $\Delta K = 0$

E. $\Delta U > 0$ and $\Delta K < 0$

(Last) (First)

5. $m = 2 \text{ kg}$ 

The surface shown in the figure is frictionless. If the block is released from rest, it will compress the spring at the foot of the incline

Hint: use the conservation of mechanical energy. You will need to solve a quadratic equation.

- A. 4.0 m
- B. 3.24 m
- C. 1.57 m
- D. 0.989 m**

The mechanical energy of the system spring + mass + Earth is constant

$$\text{Initially: } E_{\text{mech}} = K + U = 0 + mgx_i \sin \theta \quad (\text{with } x_i = 4\text{m and } \theta = 30^\circ)$$

$$\text{Finally: } E_{\text{mech}} = K + U = 0 + mgx_f \sin \theta + \frac{1}{2} kx_f^2$$

$$\text{Solve } mgx_f \sin \theta + \frac{1}{2} kx_f^2 = mgx_i \sin \theta$$

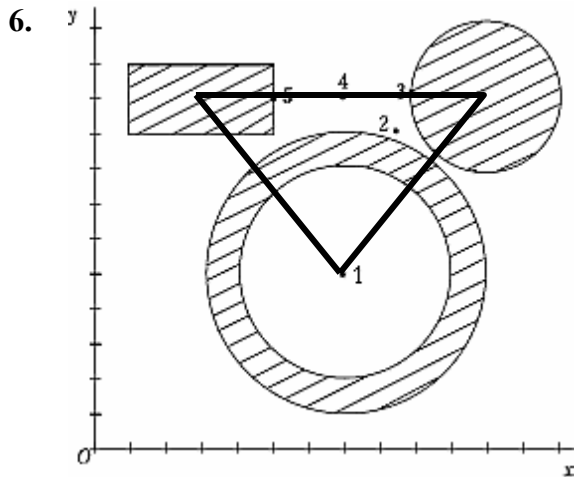
$$9.8x_f + 50x_f^2 = 39.2 \Rightarrow x_f = -0.99\text{m or } +0.79\text{m}$$

Only the negative root makes sense since the spring must be compressed.

- E. 0.5 m

(Last)

(First)



The mass of the rectangle in the figure is M , the mass of the ring is M , and the mass of the circle is $3M$. The center of mass of the system (consisting of the rectangle, the circle and the ring) with respect to the origin O is located at point

- A. 1
2

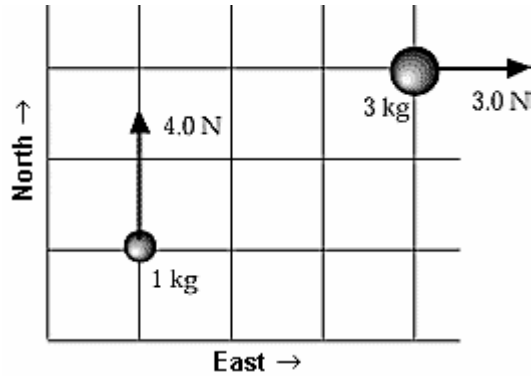
B. The center of mass must be inside the triangle linking the 3 centers of masses of the three objects. Since the circle is more massive ($3M$ as compared to M), the center of mass should be closer to the circle than to the other two objects.

- C. 3
D. 4
E. 5

(Last)

(First)

7.



A 1.0-kg mass is acted on by a net force of 4.0 N and a 3.0-kg mass is acted on by a net force of 3.0 N, in the directions shown. The acceleration of the center of mass of this system is approximately

1.25 m/s², 53° N of E

$$\vec{F}_{net} = m\vec{a} = 4\vec{a}$$

A. Also: $\vec{F}_{net} = 4\hat{y} + 3\hat{x}$, thus $a = \frac{\sqrt{4^2 + 3^2}}{4} = 1.25 \text{ m/s}^2$

The angle with the x axis is $\tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$

B. 1.85 m/s², 45° N of E

C. 4.00 m/s², due north

D. 5.30 m/s², N of E

E. 7.25 m/s², 53° N of E

(Last) (First)

8. A boy and girl initially at rest on ice skates face each other. The girl has a mass of 20 kg and the boy has a mass of 30 kg. The boy pushes the girl backward at a speed of 3.0 m/s. As a result of the push, the speed of the boy is

- A. 0 m/s
 B. 2.0 m/s

Use conservation of momentum:

B.
$$\vec{p}_{boy} + \vec{p}_{girl} = 0 \Rightarrow |\vec{v}_{boy}| = \frac{m_{girl}}{m_{boy}} |\vec{v}_{girl}| = 2 \text{ m/s}$$

- C. 3.0 m/s
 D. 4.5 m/s
 E. 9.0 m/s

9. If a body moves in such a way that its linear momentum is constant, then

- A. its kinetic energy is zero.
 B. the sum of all the forces acting on the body is constant and nonzero.
 C. its acceleration is greater than zero and is constant.
 D. its center of mass remains at rest.
E. the sum of all the forces acting on it must be zero.

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = 0$$

10. While in horizontal flight at a speed of 20 m/s, a baseball of mass 0.11 kg is struck by a bat. After leaving the bat, the baseball has a speed of 29 m/s in a direction opposite to its original direction. The magnitude of the impulse given the ball is

- A. 0.99 kg · m/s
 B. 3.2 kg · m/s
 C. 2.2 kg · m/s
D. 5.4 kg · m/s

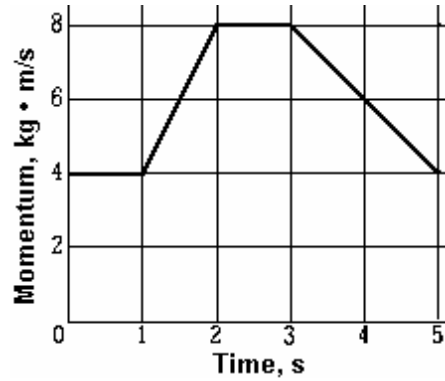
$$\vec{J} = \Delta\vec{p} = m(\vec{v}_f - \vec{v}_i) = 0.11 \times (29\hat{x} - (-20\hat{x})) = 5.39\hat{x} \text{ kg}\cdot\text{m/s} \quad \text{where } \hat{x} \text{ is a unit vector in the direction of } \vec{v}_f$$

- E. 0.55 kg · m/s

(Last)

(First)

11.



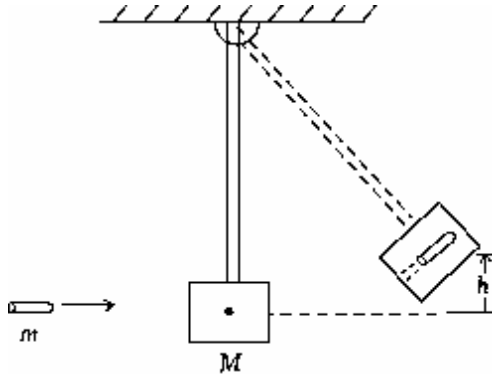
The graph shows the momentum of a body as a function of time. The time at which the force acting on the body is greatest is

- A. 0.5 s
- B. 2.5 s
- C. 4.0 s
- D. 1.5 s**

$\vec{F} = \frac{d\vec{p}}{dt}$ The force is maximum where the slope of $p(t)$ is maximum.

- E. 5.0 s

12. (Last) (First)



A bullet, $m = 0.500$ kg, traveling with a velocity v strikes and embeds itself in the bob of a ballistic pendulum, $M = 9.50$ kg. The combined masses rise to a height $h = 1.28$ m. The velocity v of the bullet is

- A. 5.00 m/s
- B. 50 m/s
- C. 100 m/s**

The velocity V of the bob + bullet just after the collision is (using conservation of momentum)

$$mv = (m + M)V \Rightarrow V = \frac{m}{m + M}v$$

Use the work energy theorem between the instant just after the collision and the instant when the bob+bullet system reaches the height h

$$0 - \frac{1}{2}(m + M)V^2 = -(m + M)gh \Rightarrow V = \sqrt{2gh}$$

It follows:

$$v = \frac{m + M}{m} \sqrt{2gh} = \frac{10}{0.5} \sqrt{2 \times 9.8 \times 1.28} = 100.2 \text{ m/s}$$

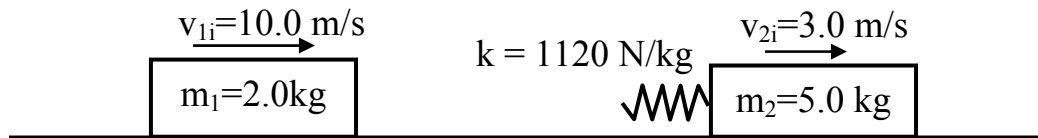
- D. 250 m/s
- E. 275 m/s

(Last)

(First)

PROBLEM [40 points]

Two masses $m_1=2.0$ kg and $m_2=5.0$ kg are on a horizontal frictionless surface. Mass m_1 is moving to the right with velocity $v_{1i} = 10$ m/s and m_2 with velocity $v_{2i} = 3.0$ m/s. As shown in the figure a massless spring of force constant $k = 1120$ N/kg is attached to m_2 .



- 1). [15 pts] After the masses collide and separate completely from one another, what are the final velocities of the two masses? (Hint: the collision is elastic).

Use the conservation of momentum and the conservation of the kinetic energy for the system made of the 2 masses.

$$\begin{cases} m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \\ \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \end{cases}$$

Using the approach described in the text p 204, we find

$$\begin{cases} m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \\ v_{1i} + v_{1f} = v_{2i} + v_{2f} \end{cases} \Rightarrow \begin{cases} 2v_{1f} + 5v_{2f} = 35 \\ v_{1f} - v_{2f} = -7 \end{cases} \Rightarrow \begin{cases} v_{1f} = 0 \text{ m/s} \\ v_{2f} = 7 \text{ m/s} \end{cases}$$

- 2). [10 pts] What are the final kinetic energies of each of the two masses (K_{1f} and K_{2f})?

$$K_{1f} = \frac{1}{2} m_1 v_{1f}^2 = 0 \text{ J}$$

$$K_{2f} = \frac{1}{2} m_2 v_{2f}^2 = 122.5 \text{ J}$$

- 3). [15 pts] What is the maximum potential energy stored in the spring during the collision? (Hint: at maximum compression, both masses have the same velocity equal to the center of mass velocity. Use also that the mechanical energy of m_1 +spring+ m_2 is constant).

$$\text{The velocity of the center of mass is } v_{cm} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{20 + 15}{7} = 5 \text{ m/s}$$

Just before the collision the mechanical energy is

$$E_{mech} = K_{1i} + K_{2i} = 122.5 \text{ J}$$

When the spring is at maximum compression, both masses have a velocity equal to the center of mass velocity. The mechanical energy is

$$E_{mech} = \frac{1}{2} m_1 v_{cm}^2 + \frac{1}{2} m_2 v_{cm}^2 + U_{el} = 87.5 + U_{el} \Rightarrow U_{el} = 35 \text{ J}$$