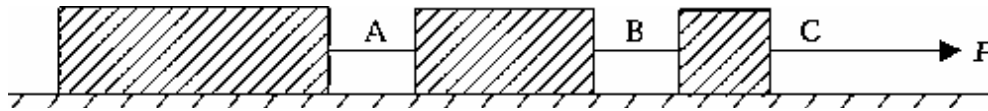


Multiple choice questions

Answer all of the following questions. Read each question carefully. **Fill the correct bubble on your scantron sheet.** Each question has exactly one correct answer. All questions are worth the same amount of points.

1.



On a frictionless horizontal surface, three boxes are connected by inextensible massless strings and are pulled by a force F as shown in the figure. The masses of the boxes are proportional to their sizes. Which string has to be the strongest so as not to break?

All 3 boxes have the same acceleration. The tension in one string is the net force applied to the set of boxes and strings to the left of that string, thus

$$T_A = m_A a$$

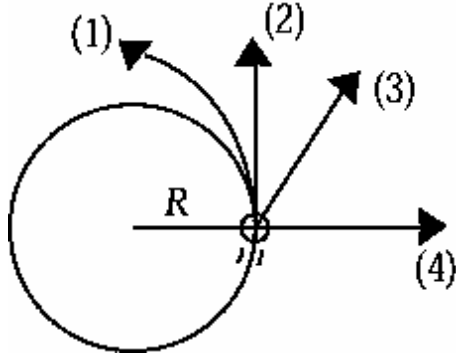
$$T_B = (m_A + m_B) a$$

$$T_C = (m_A + m_B + m_C) a$$

T_C is the largest.

- A. A
- B. B
- C. C
- D. A, B and C must be equally strong
- E. A and B must be equally strong.

2.



The figure shows a *top view* of a ball on the end of a string traveling counterclockwise in a circular path. The speed of the ball is constant. If the string should break at the instant shown, the path that the ball would follow is

A. 1

B. 2

After the string broke, the ball is free falling. Its path is a parabola which viewed from the top is a straight line tangent to the velocity of the ball when the string breaks.

C. 3

D. 4

E. impossible to tell from the given information

3. A 5-kg object undergoes a displacement $\Delta\vec{s} = -5\hat{i} + 2\hat{j}$. During the displacement, a constant force $\vec{F} = 3\hat{i} + 8\hat{j}$ acts on the object. All values are given in SI units. The work done by the force \vec{F} on this object is (in Joules)

A. -15 J

B. 16

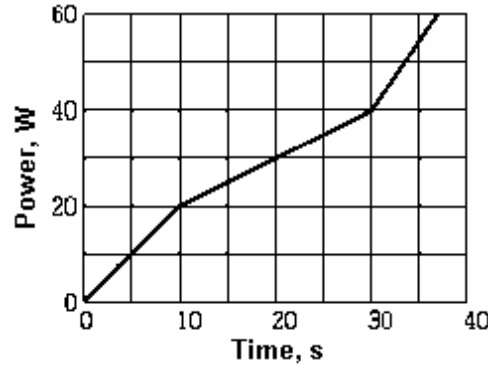
C. -34

D. 1

$$W = \vec{F} \cdot \Delta\vec{s} = 3 \times (-5) + 8 \times 2 = 1$$

E. 6

4.



A motor develops power as shown in the graph. The energy expended by the motor in the time interval between $t = 10$ s and $t = 30$ s is

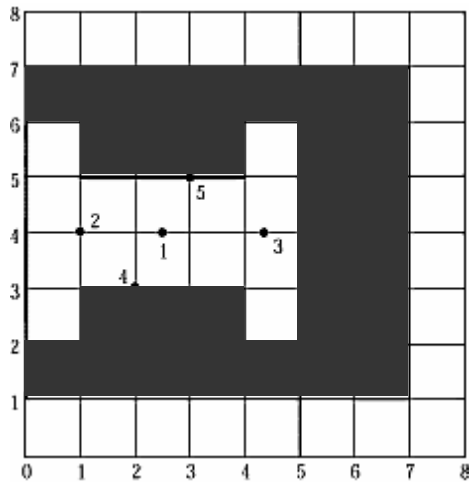
- A. 1.0 J
- B. 1.3 J
- C. 0.20 kJ
- D. 0.60 kJ**

since $P = dW/dt$,

$$W = \int_{10s}^{30s} P dt = 600J \text{ (the area under the power curve between 10 and 30s).}$$

- E. 0.70 kJ

5.



The shaded area in the figure represents a uniformly thick sheet of metal. The center of mass of the sheet is closest to point

- A. 1
- B. 2
- C. 3**

By symmetry the CM is along the 213 line.

1 is the CM of the top and bottom parts of the sheet of metal.

Taking into account the right part, the CM moves to the right to point 3.

- D. 4
- E. 5

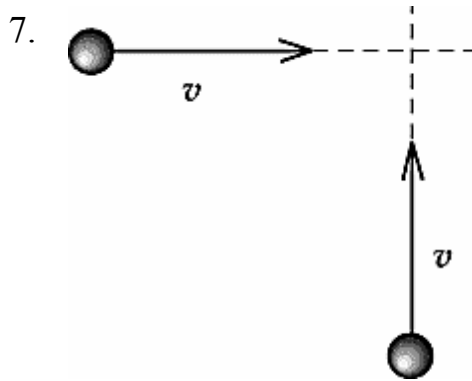
6. You shoot an arrow with a mass of 0.54 kg from a bow. The bow exerts a force of 125 N for 0.65s. The speed of the arrow as it leaves the bow is

- A. 0.10 km/s
- B. 0.15 km/s**

$$\Delta \vec{p} = \int \vec{F} dt$$

$$v_{final} = \frac{F \Delta t}{m} = \frac{125 \times 0.65}{0.54} = 150 \text{ m/s}$$

- C. 0.23 km/s
- D. 0.27 km/s
- E. 0.30 km/s



Two identical bodies of mass M move with equal speeds v . The direction of their velocities is illustrated in the figure. The magnitude of the linear momentum of the system is

In an (x,y) frame such the x axis is directed left to right and the y axis bottom to top,

A. $2Mv$

B. Mv

C. $4Mv$

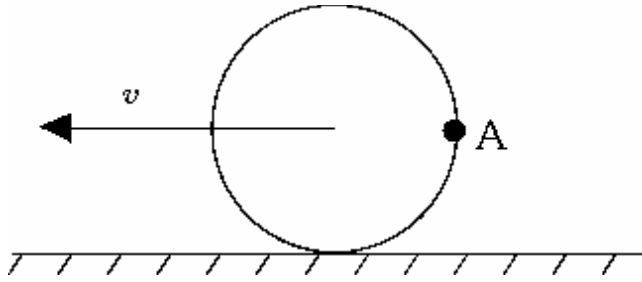
D. $\sqrt{2}Mv$

$$\vec{p}_{total} = \sum \vec{p} = Mv\hat{x} + Mv\hat{y}$$

$$|\vec{p}_{total}| = \sqrt{(Mv)^2 + (Mv)^2} = \sqrt{2}Mv$$

E. $4\sqrt{2}Mv$

8.



A bicycle is moving at a speed $v = 12.6$ m/s. The wheels of the bicycle roll without slipping. A small stone is stuck to one of the tires. At the instant the stone is at point A in the figure, it comes free. The velocity of the stone (magnitude and direction) relative to the earth just after release is

- A.** 17.8 m/s at 45° above the horizontal, toward the front of the bicycle.
 Work in an (x,y) frame: x is directed to the left, and y is directed up

The angular velocity of the wheel is

$$\omega = \frac{v}{r}$$

In the center of mass frame of the wheel the velocity of A is

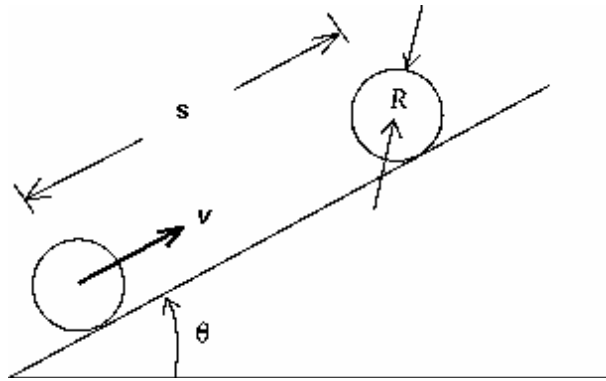
$$\vec{v}_{A/CM} = \omega r \hat{y} = v \hat{y}$$

Relative to the Earth the velocity of the wheel is

$$\vec{v}_{A/Earth} = \vec{v}_{A/CM} + \vec{v}_{CM/Earth} = v \hat{y} + v \hat{x}$$

- B. 12.6 m/s at 45° above the horizontal, away from the bicycle.
 C. 12.6 m/s at 37° below the horizontal.
 D. 12.6 m/s straight up.
 E. 17.8 m/s at 45° above the horizontal, toward the back of the bicycle.

9.



A solid disk ($I_{\text{cm}} = \frac{1}{2} mR^2$) rolls without slipping up a plane a distance s . The plane is inclined at an angle θ with the horizontal. The disk has mass m , radius R , and an initial translational speed v . How far does the disk travel up the incline?

A. $\frac{3v^2}{4g \sin \theta}$

Use the work energy theorem

$$KE_i = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{3}{4}mv^2$$

$$KE_f = 0$$

$$W_{\text{net}} = W_{\vec{w}_{DE}} = -mgs \sin \theta$$

$$KE_f - KE_i = W_{\text{net}} \Rightarrow s = \frac{3v^2}{4g \sin \theta}$$

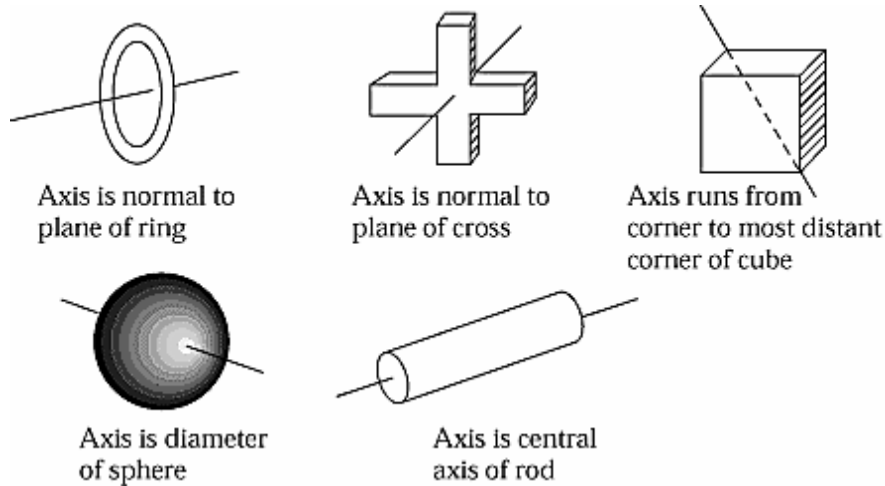
B. $\frac{v^2}{2g \sin \theta}$

C. $\frac{Rv}{2g \sin \theta}$

D. $\frac{v^2}{2g(\sin \theta - \cos \theta)}$

E. $\frac{v^2}{g \sin \theta}$

10.



Assume that none of the objects is hollow.

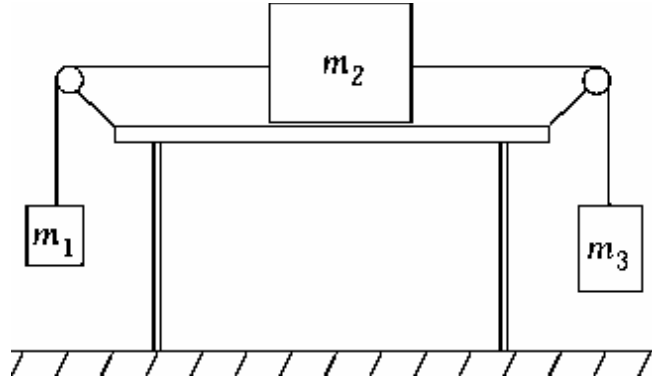
If all of the objects illustrated in the figure have equal masses, which object has the largest moment of inertia about the indicated axis?

Hint: for this question, you don't need to know the formulas giving the moments of inertia of these particular solids. Just use the definition of the moment of inertia: $I = \sum m_i r_i^2$

- A.** ring
Imagine manufacturing all of these solids from the same block of material. To maximize the moment of inertia, one needs to put the parts making up the object as far as possible from the axis of rotation. It is the ring that allows one to place all of the parts the farthest from the axis of rotation.
- B. cross
- C. sphere
- D. cube
- E. rod

Questions 11 through 17 all refer to the same problem.

For this problem, assume no friction. A mass $m_2 = 3.5$ kg rests on a horizontal table and is attached by strings to masses $m_1 = 1.5$ kg and $m_3 = 2.5$ kg as shown. The masses m_1 and m_3 hang freely.
 $g = 9.8 \text{ m/s}^2$



11. Initially the system of the 3 masses is held at rest by a hand that exerts a normal force on m_2 (the hand is not represented on the drawing above).

If \vec{T}_1 is the tension by the string on m_1 and \vec{T}_3 is the tension by the string on m_3 , how do the magnitudes of the tensions compare when the system is held at rest?

- A. $|\vec{T}_1| > |\vec{T}_3|$
B. $|\vec{T}_1| = |\vec{T}_3|$
C. $|\vec{T}_1| < |\vec{T}_3|$

When at rest, the tension force applied on m_1 or m_3 is equal to the weight of m_1 or m_3 . And we know $m_1 < m_3$

- D. Can't tell. There is not enough information.

At $t=0$, the hand is removed from m_2 and the 3 masses start moving from rest.

12. How do the magnitudes of the tensions \vec{T}_1 and \vec{T}_3 compare when the system is moving?

A. $|\vec{T}_1| > |\vec{T}_3|$

B. $|\vec{T}_1| = |\vec{T}_3|$

C. $|\vec{T}_1| < |\vec{T}_3|$

Since $m_1 < m_3$, m_1 moves up, m_2 moves to the right and m_3 moves down.

The system starts from rest, thus the acceleration of m_2 is to the right. Along an horizontal axis oriented left to right, Newton's 2nd law applied to m_2 reads:

$$T_3 - T_1 = m_2 a \text{ with } a > 0$$

- D. Can't tell. There is not enough information.

13. When the system is moving, what is the acceleration of mass m_2 (in m/s^2)?

- A. 0
- B. 1.3**

Apply Newton's 2nd law to m_1 (along an axis directed up), to m_2 (as done above), and to m_3 (along an axis directed down)

$$\left. \begin{array}{l} T_1 - m_1 g = m_1 a \\ T_2 - T_1 = m_2 a \\ m_3 g - T_2 = m_3 a \end{array} \right\} \Rightarrow (m_3 - m_1)g = (m_1 + m_2 + m_3)a$$

$$a = \frac{1}{7.5} \times 9.8 = 1.3 \text{ m/s}^2$$

- C. 5.2
- D. 8.7
- E. 9.8

14. At instant t_1 , m_3 has moved by 1m from its initial position. From the initial position to the position at t_1 , how does the work done on m_3 by the weight of m_3 , $W_{\vec{w}_{3E}}$, compare with the work done on m_1 by the weight of m_1 , $W_{\vec{w}_{1E}}$?

A. $|W_{\vec{w}_{3E}}| > |W_{\vec{w}_{1E}}|$

If h is the distance by which m_3 has moved: $h=1\text{m}$

\vec{w}_{3E} does positive work as m_3 moves down: $W_{\vec{w}_{3E}} = m_3 gh$

\vec{w}_{1E} does negative work as m_1 moves up: $W_{\vec{w}_{1E}} = -m_1 gh$

We know that $m_1 < m_3$

B. $|W_{\vec{w}_{3E}}| < |W_{\vec{w}_{1E}}|$

C. $W_{\vec{w}_{3E}} = W_{\vec{w}_{1E}}$

D. $W_{\vec{w}_{3E}} = -W_{\vec{w}_{1E}}$

E. Can't tell. There is not enough information.

15. At t_1 , what is the kinetic energy of the three masses (in Joules)?

A. 0

B. 9.8

Apply the work energy theorem

For the system $m_1+m_2+m_3$,

$$\left. \begin{aligned} KE_f &= \frac{1}{2}(m_1 + m_2 + m_3)v^2 \\ KE_i &= 0 \\ W_{net} &= W_{\vec{w}_{3E}} + W_{\vec{w}_{1E}} = (m_3 - m_1)gh \end{aligned} \right\} \Rightarrow KE_f = W_{net} = 9.8J$$

C. 19.6

D. 29.4

E. 39.2

16. At $t=0s$, the total mechanical energy of the system of the three masses and strings is 0J, what is its total mechanical energy at t_1 (in Joules)?

A. 0

All forces that do work are conservative (namely the weights of m_1 and m_3).

B. 9.8

C. 39.2

D. -9.8

E. -39.2

17. What is the value of t_1 (in seconds)?

A. 1

B. 1.2

at t_1 , m_3 has moved by $h=1m$

$$h = \frac{1}{2}at_1^2 \Rightarrow t_1 = \sqrt{\frac{2h}{a}} = 1.24s$$

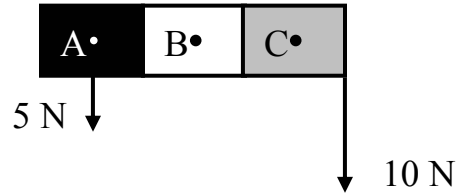
C. 1.5

D. 1.8

E. 2.1

Questions 18 through 21 all refer to the same problem.

A bar is constructed from three equally sized pieces made of different materials as shown below. It has a total mass of $m=3\text{kg}$. The points A, B and C are located at the center of mass of each piece.



The bar is initially at rest on a frictionless horizontal surface. When forces of 5N and 10N are applied at the locations shown, the bar begins to rotate counterclockwise about its center of mass.

18. What can you say about the location of the center of mass of the bar?
Choose the statement that is the most precise.

- A. It is to the left of B
- B. It is at B
- C. It is to the right of B

Since the bar starts rotating counterclockwise from rest, the net torque about the CM of the bar is counterclockwise (out of the page). The distance between the 5N force and the CM must be at least twice as large as the distance between the CM and the 10N force.

- D. It is at A
- E. It is between A and B

19. What is the direction of the acceleration of the center of mass?

A.



The net force acting on the bar is directed down (sum of the 5N and 10N forces)

B.



C.



D.



E. undefined since it is 0

20. What is the magnitude of the acceleration of the center of mass (in m/s^2)?

A. 0

B. 1.67

C. 3.33

D. 5

Use Newton's 2nd law:

$$10 + 5 = 3a \Rightarrow a = 5 m/s^2$$

E. 15

21. In a second experiment, the forces applied on the bar are doubled (the 5N force becomes 10N and the 10N force becomes 20N). Nothing else changes.

How would the acceleration \vec{a}_{CM} of the center of mass and the angular acceleration $\vec{\alpha}_{bar}$ of the bar compare between the first and the second experiments?

A. $|\vec{a}_{CM}^{exp2}| = |\vec{a}_{CM}^{exp1}|$ and $|\vec{\alpha}_{CM}^{exp2}| = |\vec{\alpha}_{CM}^{exp1}|$
 $|\vec{a}_{CM}^{exp2}| = 2|\vec{a}_{CM}^{exp1}|$ and $|\vec{\alpha}_{CM}^{exp2}| = 2|\vec{\alpha}_{CM}^{exp1}|$

B. The net force and the net torque increase by a factor of 2. Thus both linear and angular accelerations increase by a factor of 2.

C. $|\vec{a}_{CM}^{exp2}| = 2|\vec{a}_{CM}^{exp1}|$ and $|\vec{\alpha}_{CM}^{exp2}| = |\vec{\alpha}_{CM}^{exp1}|$

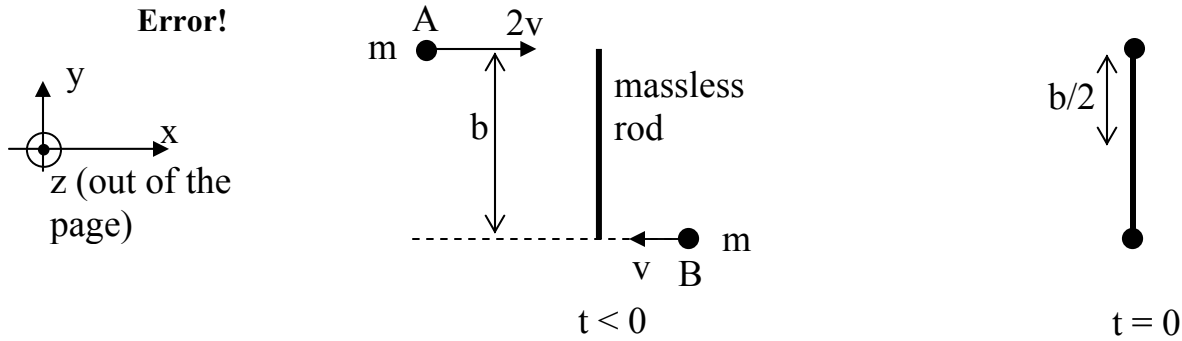
D. $|\vec{a}_{CM}^{exp2}| = |\vec{a}_{CM}^{exp1}|$ and $|\vec{\alpha}_{CM}^{exp2}| = 2|\vec{\alpha}_{CM}^{exp1}|$

E. $|\vec{a}_{CM}^{exp2}| = \frac{1}{2}|\vec{a}_{CM}^{exp1}|$ and $|\vec{\alpha}_{CM}^{exp2}| = \frac{1}{2}|\vec{\alpha}_{CM}^{exp1}|$

Questions 22 through 27 all refer to the same problem.

On a frictionless horizontal table, one puck A of mass m moves with speed $2v$ to the right. An identical puck B moves with speed v to the left as indicated on the figure below.

At $t=0$ they collide with a massless rod and stick to the ends of the rod.



22. What is the linear momentum of the system rod +A +B at $t < 0$?

- A. $2mv\hat{x}$
- B.** $mv\hat{x}$
 $p_{total} = 2mv\hat{x} - mv\hat{x} = mv\hat{x}$
- C. $-mv\hat{x}$
- D. 0
- E. $3mv\hat{x}$

23. What is the linear momentum of the system rod+A+B at $t > 0$?

- A. $2mv\hat{x}$
- B.** $mv\hat{x}$
 The net force on the system A+B+rod is 0. The total linear momentum is conserved.
- C. $-mv\hat{x}$
- D. 0
- E. $3mv\hat{x}$

24. What is the angular momentum of the system rod+A+B just before the collision in the center of mass frame of the system rod+A+B? (\hat{z} is a unit vector out of the page on the figures above)

Hint: recall that for a point particle of mass m and velocity \vec{v} , $\vec{L} = \vec{r} \times m\vec{v}$ and that for a system of particles $\vec{L}_{total} = \sum \vec{L}$

- A. $2bmv\hat{z}$
- B. $\frac{3}{2}bmv\hat{z}$
- C. 0
- D. $-2bmv\hat{z}$
- E.** $-\frac{3}{2}bmv\hat{z}$

The velocity of the center of mass is

$$\vec{v}_{CM} = \frac{2mv\hat{x} - mv\hat{x}}{3m} = \frac{1}{3}mv\hat{x}$$

$$\vec{L}_{total} = \frac{b}{2}\hat{y} \times m\left(2v\hat{x} - \frac{1}{3}v\hat{x}\right) + \left(-\frac{b}{2}\hat{y}\right) \times m\left(-v\hat{x} - \frac{1}{3}v\hat{x}\right) = -\frac{3}{2}bmv\hat{z}$$

25. What is the angular momentum of the system rod+A+B just after the collision in the center of mass frame of the system rod+A+B? (\hat{z} is a unit vector out of the page on the figures above)

Hint: what is the net torque acting on the system rod + A + B?

- A. $2bmv\hat{z}$
- B. $\frac{3}{2}bmv\hat{z}$
- C. 0
- D. $-2bmv\hat{z}$
- E.** $-\frac{3}{2}bmv\hat{z}$

There is no net torque on the system rod + A + B. The angular momentum of the system is conserved.

26. After the collision, what is the moment of inertia of the system rod+A+B about an axis of rotation going through its center of mass and parallel to \hat{z} ?

A. $\frac{1}{4}mb^2$

B. $\frac{1}{2}mb^2$

$$I = m\left(\frac{b}{2}\right)^2 + m\left(\frac{b}{2}\right)^2 = \frac{1}{2}mb^2$$

C. mb^2

D. $2mb^2$

E. $4mb^2$

27. After the collision, what is the magnitude ω of the angular velocity of the system rod+A+B about its center of mass?

A. 0

B. $\frac{v}{2b}$

C. $\frac{2v}{b}$

D. $\frac{3v}{b}$

$$L = I\omega \Rightarrow \omega = \frac{3bmv}{2\frac{1}{2}mb^2} = \frac{3v}{b}$$

E. $\frac{4v}{b}$