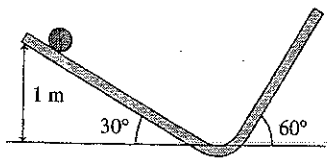
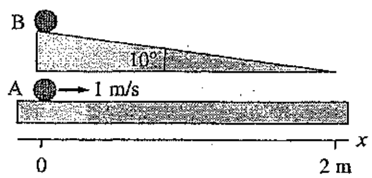
Example 1: Bob throws a ball straight up at 20 m/s, releasing the ball 1.5 m above the ground. What is the maximum height of the ball? What is the ball's impact speed as it hits the ground?

Example 2: A ball is released at a height of 1.0 m on a frictionless 30° slope. At the bottom, it turns smoothly onto a 60° slope going back up. What maximum height does it reach on the right side?

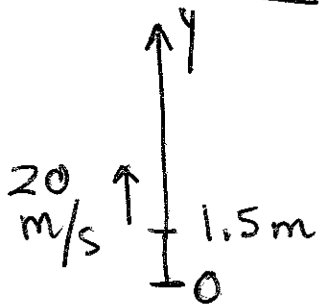


Example 3: A sprinter accelerates at 2.5 m/s^2 until reaching his top speed of 15 m/s. He then continues to run at top speed. How long does it take him to run the 100-m dash?

Example 4: Ball A rolls along a frictionless, horizontal surface at a speed of 1.0 m/s. Ball B is released from rest at the top of a 2.0-m-long, 10° ramp at the exact instant ball A passes by. Will B overtake A before reaching the bottom of the ramp? If so, at what position?



Example1



- at the top, $v_y = 0$

Use $v_{yf}^2 - v_{yi}^2 = 2a_y(y_f - y_i)$

with $v_{yf} = 0$, $v_{yi} = 20\ m/s$

$a_y = -9.8\ m/s^2$

$y_f = ?$ $\quad y_i = 1.5\ m.$

$$0^2 - 20^2 = 2(-9.8)(y_f - 1.5)$$

$\boxed{y_f = 21.9\ m}$ (maximum height above ground)

- Use again $v_{yf}^2 - v_{yi}^2 = 2a_y(y_f - y_i)$

with $v_{yf} = ?$ $\quad v_{yi} = 20\ m/s$ $\quad a_y = -9.8\ m/s^2$

$y_f = 0\ m$ $\quad y_i = 1.5\ m$

$$v_{yf}^2 - 20^2 = 2(-9.8)(0 - 1.5)$$

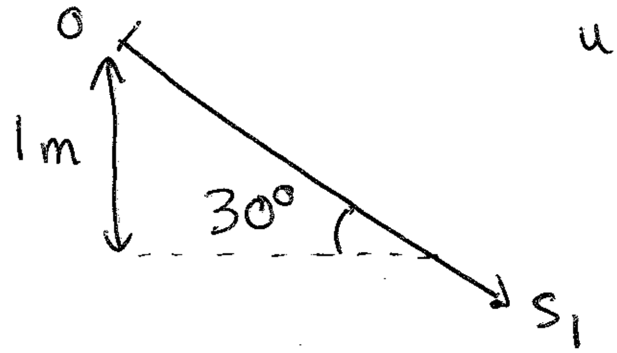
$$v_{yf}^2 = 429.4\ m^2/s^2$$

On impact, obviously $v_{yf} < 0$

so $\boxed{v_{yf} = -20.7\ m/s}$

Example 2

- Ball rolling down:



use $v_{f_1}^2 - v_{i_1}^2 = 2a_1(s_{f_1} - s_{i_1})$

with $v_{i_1} = 0$ m/s

$v_{f_1} = ?$

$a_1 = 9.8 \sin 30°$.

$s_{i_1} = 0$.

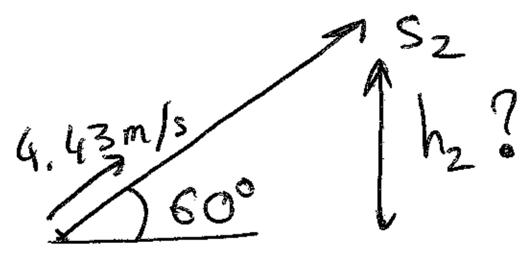
$s_{f_1} = \dfrac{1m}{\sin 30°} = 2m$

$v_{f_1}^2 = 0^2 + 2 \cdot 9.8 \cdot \sin 30\,(2-0)$

$v_{f_1}^2 = 19.6\ m^2/s^2$

$v_{f_1} = 4.43$ m/s

- Ball rolling up



at the top, $v_{f_2} = 0$

$$v_{f_2}^2 - v_{i_2}^2 = 2a_2(s_{f_2} - s_{i_2})$$

$v_{i_2} = 4.43$ m/s, $a_2 = -9.8 \sin 60$, $s_{i_2} = 0$ m

$s_{f_2} = \dfrac{h_2}{\sin 60°}$

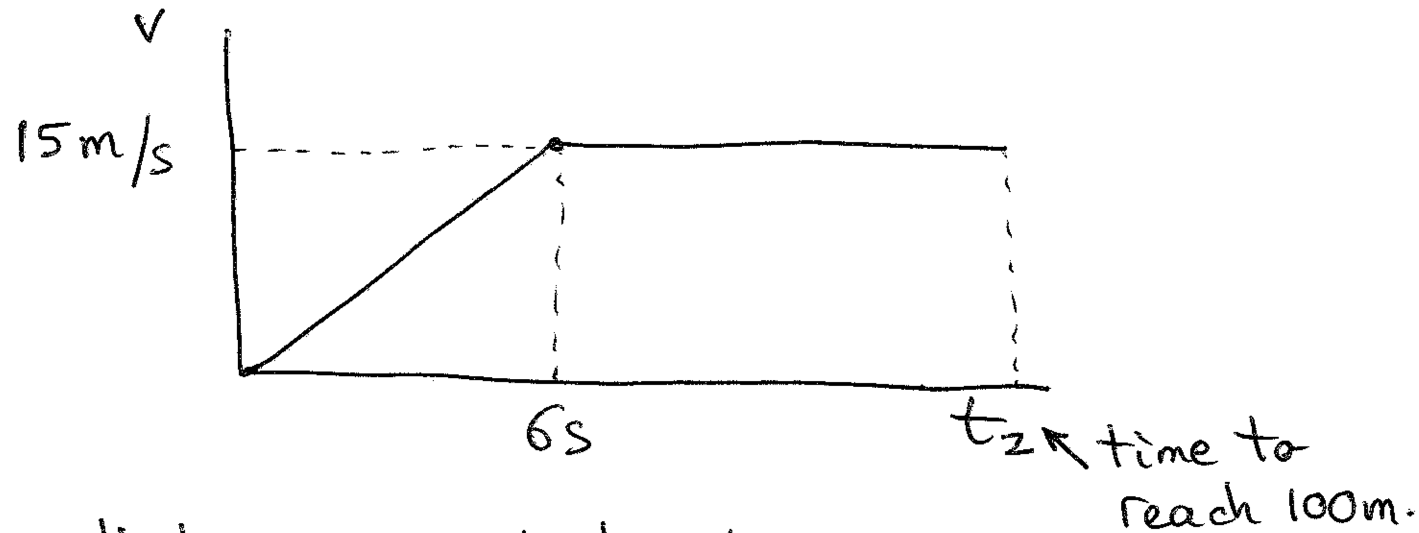
$$0^2 - 19.6 = -2 \cdot 9.8 \cdot \sin 60°\left(\frac{h_2}{\sin 60°} - 0\right)$$

$\boxed{h_2 = 1\ m}$ comes back to the same height.

Example 3

To get to 15 m/s, it takes a time t_1 such that: $2.5\,t_1 = 15 \Rightarrow t_1 = 6\,s$

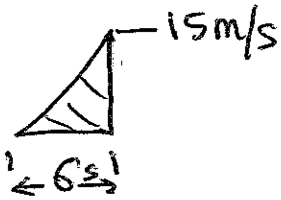
Graph v versus time:

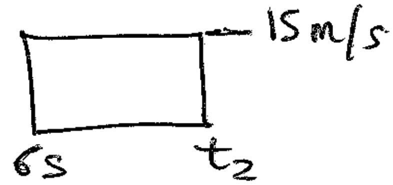


The distance covered by the sprinter is the area under the v(t) curve.

$$100\,m = \frac{15 \cdot 6}{2} + \underbrace{15 \cdot (t_2 - 6)}_{\text{area of}}$$

area of





$$t_2 = \frac{145}{15}$$

$$\boxed{t_2 = 9.67\,s}$$

Example 4.

Need to compare x_A and x_B

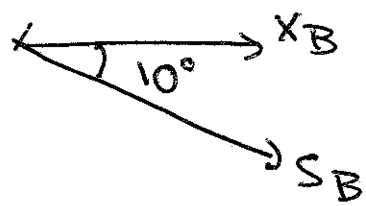
- $x_A = v_A t = t$ (v_A = 1 m/s)

- Motion of B:

$$s_B = \frac{1}{2} g \sin(10°) t^2$$

But

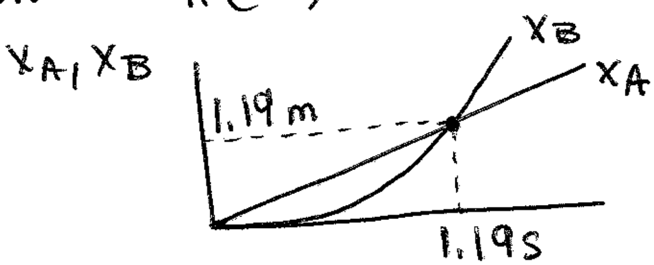
$$s_B \cos 10° = x_B$$



so $$x_B = \frac{1}{2} g \sin 10 \cos 10 \cdot t^2$$

$$x_B = 0.838\, t^2$$

Graph $x_A(t)$ and $x_B(t)$.



B passes A at $t = 1.19$ s when A and B have an $x = 1.19$ m.