CSC 143 Java

Trees

Overview

- Topics
  - Trees: Definitions and terminology
  - Binary trees
  - Tree traversals
  - Binary search trees
  - Applications of BSTs

Trees

- Most of the structures we've looked at so far are linear
  - Arrays
  - Linked lists
- There are many examples of structures that are not linear, e.g. hierarchical structures
  - Organization charts
  - Book contents (chapters, sections, paragraphs)
  - Class inheritance diagrams
- *Trees* can be used to represent hierarchical structures

Looking Ahead To An Old Goal

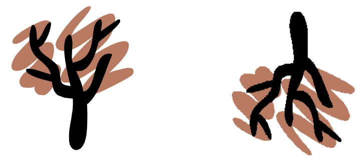
- Finding algorithms and data structures for fast searching
  - A key goal
  - Sorted arrays are faster than unsorted arrays, for searching
    - Can use binary search algorithm
    - Not so easy to keep the array in order
  - LinkedLists were faster than arrays (or ArrayLists), for insertion and removal operations
    - The extra flexibility of the "next" pointers avoided the cost of sliding
    - But... LinkedLists are hard to search, even if sorted
- Is there an analogue of LinkedLists for sorted collections??
- The answer will be...Yes: a particular type of *tree*!

Tree Definitions

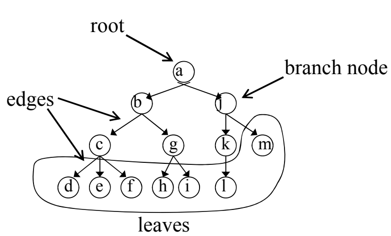
- A *tree* is a collection of *nodes* connected by *edges*
- A *node* contains
  - Data (e.g. an Object)
  - References (edges) to two or more *subtrees* or *children*
- Trees are hierarchical
  - A node is said to be the *parent* of its *children* (subtrees)
  - There is a single unique *root* node that has no parent
  - Nodes with no children are called *leaf nodes*
  - Nodes with at least one child are *branch nodes*
  - A tree with no nodes is said to be *empty*

Drawing Trees

- For whatever reason, computer sciences trees are normally drawn upside down: root at the top

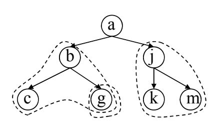


Tree Terminology



Subtrees

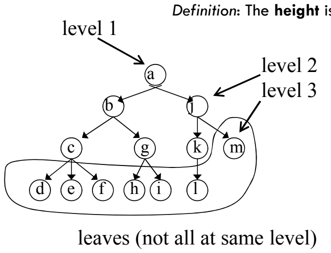
- A *subtree* in a tree is any node in the tree together with all of its descendants (its children, and their children, recursively)



- Note: note every sub*set* is a sub*tree*!

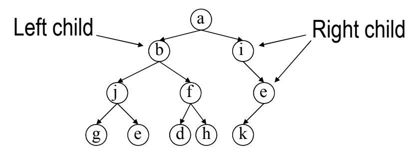
Level and Height

Definition: The root has **level** 1
Children have level 1 greater than their parent
Definition: The **height** is the highest level of a tree.



Binary Trees

- A *binary tree* is a tree each of whose nodes has no more than two children
  - The two children are called the *left child* and *right child*
  - The subtrees belonging to those children are called the *left subtree* and the *right subtree*



Binary Tree Implementation

- A node for a binary tree holds the item and references to its subtrees

```
public class BTNode<E> {
    public E item;                  // data item in this node
    public BTNode left;             // left subtree, or null if none
    public BTNode right;            // right subtree, or null if none
    public BTNode(E item, BTNode left, BTNode right) { … }
}
```

- The whole tree can be represented just by a pointer to the root node, or null if the tree is empty

```
public class BinaryTree {
    private BTNode root;            // root of tree, or null if empty
    public BinTree( ) { this.root = null; }
    …
}
```


Tree Algorithms

- The definition of a tree is naturally recursive:
  - A tree is either null,
    or data + left (sub-)tree + right (sub-)tree
  - Base case(s)?
  - Recursive case(s)?
- Given a recursively defined data structure, recursion is often a very natural technique for algorithms on that data structure
  - Don't fight it!

A Typical Tree Algorithm: size( )

```
public class BinTree {
    …
    /** Return the number of items in this tree */
    public int size( ) {
        return subtreeSize(root);
    }
    // Return the number of nodes in the (sub-)tree with root n
    private int subtreeSize(BTNode n) {
        if (n == null) {
            return 0;
        } else {
            return 1 + subtreeSize(n.left) + subtreeSize(n.right);
        }
    }
```

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Tree Traversal

- Functions like subtreeSize systematically "visit" each node in a tree
  - This is called a *traversal*
  - We also used this word in connection with lists
- Traversal is a common pattern in many algorithms
  - The processing done during the "visit" varies with the algorithm
- What order should nodes be visited in?
  - Many are possible
  - Three have been singled out as particularly useful for binary trees: *preorder*, *postorder*, and *inorder*

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Traversals

- **Preorder** traversal:
  - "Visit" the (current) node first
    i.e., do what ever processing is to be done
  - Then, (recursively) do preorder traversal on its children, left to right
- **Postorder** traversal:
  - First, (recursively) do postorder traversals of children, left to right
  - Visit the node itself last
- **Inorder** traversal:
  - (Recursively) do inorder traversal of left child
  - Then visit the (current) node
  - Then (recursively) do inorder traversal of right child

  Footnote: pre- and postorder make sense for all trees; inorder only for binary trees

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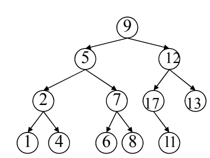
Example of Tree Traversal

In what order are the nodes visited, if we start the process at the root?



Preorder: 9, 5, 2, 1, 4, 7, 6, 8, 12, 17, 11, 13

Inorder: 1, 2, 4, 5, 6, 7, 8, 9, 17, 11, 12, 13

Postorder: 1, 4, 2, 6, 8, 7, 5, 11, 17, 13, 12, 9

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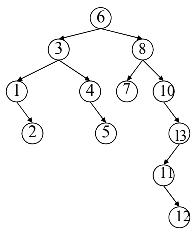
More Practice

What about this tree?



Preorder: 6, 3, 1, 2, 4, 5, 8, 7, 10, 13, 11, 12

Inorder: 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13

Postorder: 2, 1, 5, 4, 3, 7, 12, 11, 13, 10, 8, 6

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New Algorithm: *contains*

- Return whether or not a value is an item in the tree

```
public class BinaryTree {
    …
    /** Return whether elem is in tree */
    public boolean contains(Object elem) {
        return subtreeContains(root, elem);
    }
    // Return whether elem is in (sub-)tree with root n
    private boolean subtreeContains(BTNode n, Object elem) {
        if (n == null) {
            return false;
        } else if (n.item.equals(elem)) {
            return true;
        } else {
            return subtreeContains(n.left, elem) || subtreeContains(n.right, elem);
        }
    }
```

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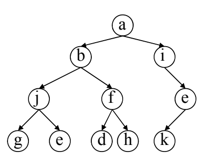
Test

contains(d)

contains(c)



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Cost of *contains*

- Work done at each node: O(1)
- Number of nodes visited: N
- Total cost: O(N)

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