CSC 143 Java

Sorting

Sorting

- Binary search is a huge speedup over sequential search
  - But requires the list be sorted
- Slight Problem: How do we get a sorted list?
  - Maintain the list in sorted order as each word is added
  - Sort the entire list when needed
- Many, many algorithms for sorting have been invented and analyzed
- Our algorithms all assume the data is already in an array
  - Other starting points and assumptions are possible

Insert for a Sorted List

- Exercise: Assume that words[0..size-1] is sorted. Place new word in correct location so modified list remains sorted
  - Assume that there is spare capacity for the new word (what kind of condition is this?)
- Before coding:
  - Draw pictures of an example situation, before and after
  - Write down the postconditions for the operation

```
// given existing list words[0..size-1], insert word in correct place and increase size
void insertWord(String word) {



    size++;
}
```


Insertion Sort

- Once we have insertWord working...
- We can sort a list in place by repeating the insertion operation

```
void insertionSort( ) {
    int finalSize = size;
    size = 1;
    for (int k = 1; k < finalSize; k++) {
        insertWord(words[k]);
    }
}
```

Insertion Sort As A Card Game Operation

- A bit like sorting a hand full of cards dealt one by one:
  - Pick up 1st card – it's sorted, the hand is sorted
  - Pick up 2nd card; *insert* it after or before 1st – both sorted
  - Pick up 3rd card; *insert* it after, between, or before 1st two
  - …
- Each time:
  - Determine where new card goes
  - Make room for the newly inserted member.

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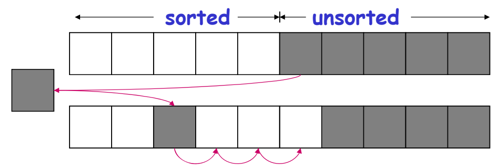
Insertion Sort As Invariant Progression



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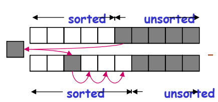
Insertion Sort Code (C++)




```
void  insert(int list[], int n) {
    int i;
    for (int j=1 ; j < n; ++j) {
        // pre: 1<=j && j<n && list[0 ... j-1] in sorted order
        int temp = list[j];
        for ( i = j-1 ; i >= 0 && list[i] > temp ; --i ) {
            list[i+1] = list[i] ;
        }
        list[i+1] = temp ;
        // post: 1<=j && j<n && list[0 ... j] in sorted order
    }
}
```

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Insertion Sort Trace

- Initial array contents

0 pear
1 orange
2 apple
3 rutabaga
4 aardvark
5 cherry
6 banana
7 kumquat

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Insertion Sort Performance

- Cost of each insertWord operation:
- Number of times insertWord is executed:
- Total cost:
- Can we do better?

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Analysis

- Why was binary search so much more effective than sequential search?
  - Answer: binary search divided the search space in half each time; sequential search only reduced the search space by 1 item
- Why is insertion sort $O(n^2)$?
  - Each insert operation only gets 1 more item in place at cost O(n)
  - O(n) insert operations
- Can we do something similar for sorting?

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Where are we on the chart?

N	$\log_2 N$	5N	$N \log_2 N$	N^2	2^N
8	3	40	24	64	256
16	4	80	64	256	65536
32	5	160	160	1024	$\sim 10^9$
64	6	320	384	4096	$\sim 10^{19}$
128	7	640	896	16384	$\sim 10^{38}$
256	8	1280	2048	65536	$\sim 10^{76}$
10000	**13**	**50000**	**10^5**	**10^8**	**$\sim 10^{3010}$**

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Divide and Conquer Sorting

- Idea: emulate binary search in some ways
  1. divide the sorting problem into two subproblems;
  2. recursively sort each subproblem;
  3. combine results
- Want division and combination at the end to be fast
- Want to be able to sort two halves independently
- This is a an algorithm strategy known as "divide and conquer"

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Quicksort

- Invented by C. A. R. Hoare (1962)
- Idea
  - Pick an element of the list: the *pivot*
  - Place all elements of the list smaller than the pivot in the half of the list to its left; place larger elements to the right
  - Recursively sort each of the halves
- Before looking at any code, see if you can draw pictures based just on the first two steps of the description

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Code for QuickSort

```
// Sort words[0..size-1]
void quickSort( ) {
    qsort(0, size-1);
}

// Sort words[lo..hi]
void qsort(int lo, int hi) {
    // quit if empty partition
    if (lo > hi) { return; }
    int pivotLocation = partition(lo, hi);     // partition array and return pivot loc
    qsort(lo, pivotLocation-1);
    qsort(pivotLocation+1, hi);
}
```

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Recursion Analysis

- Base case? Yes.

```
// quit if empty partition
if (lo > hi) { return; }
```

- Recursive cases? Yes

```
qsort(lo, pivotLocation-1);
qsort(pivotLocation+1, hi);
```

  - Observation: recursive cases work on a smaller subproblem, so algorithm will terminate

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A Small Matter of Programming

- *Partition* algorithm
  - Pick pivot
  - Rearrange array so all smaller element are to the left, all larger to the right, with pivot in the middle
- *Partition* is not recursive
- Fact of life: *partition* is tricky to get right
- How do we pick the pivot?
  - For now, keep it simple – use the first item in the interval
  - Better strategies exist

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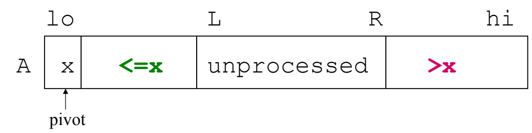
Partition design

- We need to partition words[lo..hi]
- Pick words[lo] as the pivot
- Picture:

A `Partition` Implementation

- Use first element of array section as the pivot
- Invariant:



Partition Algorithm: PseudoCode

The two-fingered method

// Partition words[lo..hi]; return location of pivot in range lo..hi
int partition(int lo, int hi)

Partition Test

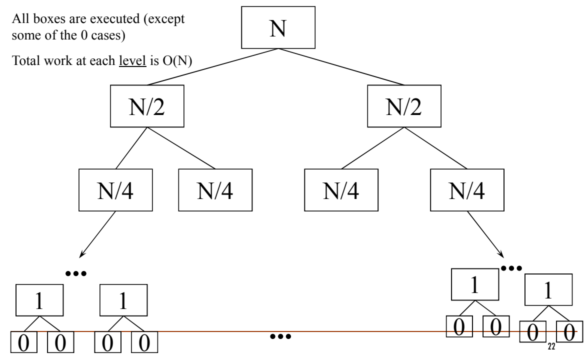
- Check: partition(0,7)

  0 orange
  1 pear
  2 apple
  3 rutabaga
  4 aardvark
  5 cherry
  6 banana
  7 kumquat

Complexity of QuickSort

- Each call to Quicksort (ignoring recursive calls):
  - One call to `partition = O(n)`, where `n` is size of *part* of array being sorted
    - Note: This n is smaller than the N of the original problem
  - Some `O(1)` work
  - Total = `O(n)` for `n` the size of array part being sorted
- Including recursive calls:
  - Two recursive calls at each level of recursion, each partitions "half" the array at a cost of `O(N/2)`
  - How many levels of recursion?

QuickSort (Ideally)



QuickSort Performance (Ideal Case)

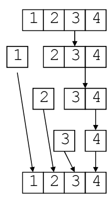
- Each partition divides the list parts in half
  - Sublist sizes on recursive calls: n, n/2, n/4, n/8….
  - Total depth of recursion: __________________
  - Total work at each level: O(n)
  - Total cost of quicksort: ________________ !
- For a list of 10,000 items
  - Insertion sort: $O(n^2)$: 100,000,000
  - Quicksort: O(n log n): 10,000 $\log_2$ 10,000 = 132,877

Best Case for QuickSort

- Assume `partition` will split array exactly in half
- Depth of recursion is then $\log_2 N$
- Total work is `O(N)*O(log N)=O(N log N)`, much better than $O(N^2)$ for selection sort
- Example: Sorting 10,000 items:
  - Selection sort: $10{,}000^2$ = 100,000,000
  - **Quicksort**: 10,000 $\log_2$ 10,000 $\approx$ **132,877**

Worst Case for QuickSort

- If we're very unlucky, then each pass through partition removes only a *single* element.



- In this case, we have N levels of recursion rather than $\log_2 N$. What's the total complexity?

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QuickSort Performance (Worst Case)

- Each partition manages to pick the largest or smallest item in the list as a pivot
  - Sublist sizes on recursive calls:
  - Total depth of recursion: ___________________
  - Total work at each level: O(n)
  - Total cost of quicksort: _________________ !

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Worst Case vs Average Case

- QuickSort has been shown to work well in the average case (mathematically speaking)
- In practice, Quicksort works well, provided the pivot is picked with some care
- Some strategies for choosing the pivot:
  - Compare a small number of list items (3-5) and pick the *median* for the pivot
  - Pick a pivot element *randomly* in the range lo..hi

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QuickSort as an Instance of Divide and Conquer

Generic Divide and Conquer	QuickSort
1. Divide	Pick an element of the list: the *pivot* Place all elements of the list smaller than the pivot in the half of the list to its left; place larger elements to the right
2. Solve subproblems separately (and recursively)	Recursively sort each of the halves
3. Combine subsolutions to get overall solution	Surprise! Nothing to do

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Another Divide-and-Conquer Sort: Mergesort

- 1. Split array in half
  - just take the first half and the second half of the array, *without* rearranging
- 2. Sort the halves separately
- 3. Combining the sorted halves ("merge")
  - repeatedly pick the least element from each array
  - compare, and put the smaller in the resulting array
  - example: if the two arrays are

    1 12 15 20

    5 6 13 21 30

    The "merged" array is

    1 5 6 12 13 15 20 21 30
  - note: we will need a temporary result array

Summary

- Recursion
  - Methods that call themselves
  - Need base case(s) and recursive case(s)
  - Recursive cases need to progress toward a base case
  - Often a very clean way to formulate a problem (let the function call mechanism handle bookkeeping behind the scenes)
- Divide and Conquer
  - Algorithm design strategy that exploits recursion
  - Divide original problem into subproblems
  - Solve each subproblem recursively
  - Can sometimes yield dramatic performance improvements